Financial Development and The Aggregate Savings Rates: A Hump-Shaped Relationship*

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Abstract

This study documents a hump-shaped empirical relationship between financial development and the aggregate savings rate across 12 Asian and 31 OECD economies. An incomplete-market model featuring both heterogeneous households and heterogeneous firms is provided to explain this hump-shaped relationship. The key insight of the model is that financial development tends to reduce the precautionary saving incentives of households but increase firms' ability to borrow and invest. As a result, the aggregate savings rate may rise initially with financial development because of greater investment by firms, but then it declines with further financial development because of substantially reduced precautionary savings by households.

Keywords: High Savings Rate Puzzle, Financial Development, Heterogenous Firms, Heterogenous Households, Borrowing Constraints

JEL Classification: E21, E22, E44

1 Introduction

Savings rates vary substantially across countries and over time. This paper investigates to what extent this large variation can be explained by the difference in financial development.[1] We focus on financial development as financial constraints seem to be a feature of reality to households as well as firms, in both developing and developed countries. More importantly, a large literature has shown that financial

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[1]There is a large literature quantitatively documents the factors such as TFP growth, demographics that determine one country's saving rate, e.g., Chen, Imrohoroglu and Imrohoroglu (2006, 2007).

constraints matter for households' consumption and savings (see e.g., Gross and Souleles (2002), Mian and Sufi (2011)). Meanwhile, numerous studies have found that financial constraints on corporate investment are pervasive even in developed economies[2]. As savings and investment constitute the supply and demand for the loanable funds market, financial development produces both demand-side and supply-side forces on the equilibrium savings rates. Financially constrained households tend to save as a precaution, so extending credit to households may lead to a decreasing household savings rate. On the other hand, financially constrained firms cannot invest to their optimal level with limited funds, so expanding credit to firms can lead to an increasing firm investment and therefore an increasing savings rate in equilibrium. Consequently, it cannot be preassumed that the effect of financial development on the savings rate will be monotonic.

Indeed, using large cross-country panel data on savings rates, we document a very robust hump-shaped relationship between financial development and aggregate savings. This hump-shaped relationship may explain why existing empirical studies produce mixed results in explaining the relationship between financial development and the savings rate. King and Levine (1993), Loayza, Schmidt-Hebbel and Serven (2000), Horioka and Yin (2010), for example, suggest a negative relationship, while Park and Shin (2009) find the impact of financial development to be insignificantly positive. These studies, however, typically presume that the relationship between financial development and savings is linear. So depending on the sample, the relationship could be either positive or negative[3].

There is surprisingly little research on the non-monotonic relationship between financial development and savings. To the best of our knowledge, the only empirical research is by Horioka and Terada-Hagiwara (2012), who document a hump-shaped relationship between financial development and savings rates in 12 Asian economies during 1996-2007. Our empirical analysis complements Horioka and Terada-Hagiwara's study in three respects. First, we expand the sample to include 31 OECD economies. It is important to confirm the non-monotonic relationship with a sample that extends beyond Asian economies, since these economies have historically had high savings rates. Second, we use various econometric methods to re-examine the statistical relationship, including both parametric and semi-parametric methods. For parametric methods, we employ both a static panel data regression, as in Horioka and Terada-Hagiwara (2012) and a dynamic panel data regression, as in Loayza, Schmidt-Hebbel and Serven (2000). Finally, we consider four additional measures of financial development, besides the private credits-to-GDP ratio used in their paper. Our analysis confirms a hump-shaped relationship between financial development and the aggregate savings rate, after controlling for conventional factors. The existence of a uniform hump-shaped relationship in a broader sample thus calls for a unified theory to explain the relationship, which is the second goal of this study.

To this end, we construct a dynamic general equilibrium model with financial constraints on both households and firms. To make the model tractable and intuitive, we borrow the household model of Wen

[2]See Hubbard (1998) for a recent survey of the earlier literature using the Q-theory of investment, and Inessa (2003) for recent international evidence from structural investment models.

[3]Case studies also show that the effects of financial development on savings are mixed. For example, Bandiera, et al. (2000) show that savings rates decline in Korea and Mexico but rise in Turkey and Ghana after financial liberalization.

(2009, 2015), which assumes a quasilinear preference. This allows us to characterize the household saving behavior in a closed form, even though households face uninsurable idiosyncratic liquidity shocks and borrowing constraints. For any given interest rate, households save excessively compared to an economy without uninsurable risks. A higher level of financial development will thus reduce the incentive to save, all else being equal. Unlike Wen (2009, 2015), firms in our model discover investment opportunities randomly as in Kiyotaki and Moore (2012), which captures the idea that investment at firm and plant levels is lumpy. Hence, only a fraction of firms invest in each period. This creates a need to transfer funds among the firms. However, frictions arise when funds are allocated between firms, due to limited enforcement. In other words, the firms are financially constrained. Assuming constant returns to scale allows us to characterize the firms' investment decision rules analytically and permits exact aggregation, so only the mean of the capital distribution matters for aggregate equilibrium.[4] We show that borrowing constraints on the firms create a gap between the return on capital and the effective real interest rate for savings. Relaxation of the borrowing constraints on firms will narrow this gap and increase the real interest rate. As a result, a high level of financial development will generate an incentive for firms to increase their investment and hence the aggregate savings rate. The overall effect on the aggregate savings rate will then depend on which side of the economy (the households or the firms) dominates.

Our model is able to generate a hump-shaped relationship between financial development and the aggregate savings rate with reasonable parameter values. To see this, imagine an extreme case in which firms have to borrow if they want to invest but they cannot borrow at all. In this case, the total investment demand would be zero. Since at equilibrium total savings must be equal to total investment, the aggregate savings rate will always be zero regardless of the households' strong incentives to save.[5] As the borrowing constraints in such an economy gradually relax (for both firms and households), the aggregate savings rate will initially go up as firms start investing heavily. Beyond a critical level, however, the downward trend in household savings will begin to dominate. Thus, further relaxation in financial constraints will reduce the aggregate savings rate. In our quantitative analysis, countries differ in three aspects. Two parameters capture the borrowing constraints faced by households and firms, respectively. To explain the large difference in GDP per capita, we assume different countries have different investment-specific technology. We focus on investment-specific technology shock as recent research shows that it may be a more dominate force in explaining the growth in output per hours worked than the neutral technology (see e.g., Greenwood, Hercowitz and Krusell (1997)). We calibrate these parameter values for different countries as follows. We first carefully calibrate the U.S. economy as a benchmark, and calibrate other economies by targeting their saving rates, private credit to GDP ratio relative to U.S., and output relative

[4] At the same time, a vast theoretical literature (e.g., Kiyotaki and Moore (1997, 2012), Bernanke, Gertler and Gilchrist (1999), Albuquerque and Hopenhayn (2004), and Cooley, Marimon, and Quadrini (2004)) has developed full dynamic general equilibrium models of investment with financial constraints to examine the aggregate implications of financial frictions for business cycle fluctuations. Our heterogenous-firm model with limited enforcement frictions and random investment opportunities is also closely related to the recent general-equilibrium-heterogeneous-firm models of Wang and Wen (2012). However, Wang and Wen (2012) do not consider heterogeneous households.

[5] Although households have strong precautionary saving incentives under large idiosyncratic risks, the effective rate of return (the interest rate) on household savings is too low to induce them to save.

to the U.S. In our calibrated exercise, we show that our calibrated model explains a substaintial variation in the saving rates, GDP per capita, and private credit to GDP ratio acorss countries. The correlation between the model simulated data and actual data is 0.54 for saving rates, 0.94 for GDP per capita, and 0.99 for the private credit to GDP ratio. Based on the model simulated cross-country dataset, we shows that our model explains the observed hump-shaped relationship in real data quite well.

We argue that borrowing constraints on both households and firms are essential to account for the hump-shaped relationship. The Buffer-Stock theory of saving (see, e.g., Kimball (1990), Deaton (1991), Carroll (1992), Aiyagari (1994)) with borrowing constraints on households typically predicts that relaxing financial constraints on households will reduce their precautionary savings, leading to a negative relationship between financial development and savings. On the other hand, models with financial constraints only on firms generally imply that more investment will be financed, producing a positive relationship.[6] As previously mentioned, there already exists a vast empirical literature that studies the interaction between financial constraints and corporate investment. To directly test these two opposite forces of financial development on the savings, we conduct an additional empirical analysis on the relationship between the savings rate and the household-side financial development measured by the volume of credits extended to the households in Appendix E. We find that the savings rate is monotonically decreasing in the household-side financial development as predicted by our model.

The rest of the paper is organized as follows. Section 2 presents the empirical evidence of a hump-shaped relationship between financial development and the aggregate savings rate. Section 3 presents a dynamic general equilibrium model in which both households and firms face financial constraints. Section 4 and 5 conduct theoretical and quantitative analysis to explain the empirical findings. Section 6 concludes.

2 Empirical Evidence

In this section, we focus on the empirical evidence of a hump-shaped relationship between financial development and the aggregate savings rate. We use various subsamples and estimators to examine the nonlinear relationship. To save space, the empirical evidence of a monotonic decreasing relationship between financial development and the household savings rate is in Appendix E.

2.1 Data

We collect annual time series data from 1960 to 2008 on the savings rate and its potential explanatory variables for the same 12 Asian economies studied by Horioka and Terada-Hagiwara (2010) and an extra group of 31 OECD economies. Because Japan and South Korea belong to both groups, the total number

[6] Examples includes the credit rationing model of Stiglitz and Weiss (1981), the costly state verification models of Townsend (1979), Gale (1985), and Bernanke and Gertler (1989), the limited contract enforcement model of Kiyotaki and Moore (1997), and the moral hazard model of Holmstrom and Tirole (1998).

of economies we study is 41 instead of 43. The size of the sample is determined by the availability of observations. In particular, in order to implement the GMM-IV estimator described below, we restrict our sample to include countries with at least 5 consecutive annual observations.

Our sample draws from the World Development Indicators of the World Bank and the Penn World Table 7.0. A detailed description of the sample is presented in Appendix A. Like Loayza, Schmidt-Hebbel and Serven (2000), we set a threshold of ±60% annual inflation rate and real interest rate to exclude episodes of high inflation in the sample, and work with the original data instead of phase-averaged data using an arbitrary phase length. The results are almost the same if we set the threshold to ±50%.

The data show that both the savings rates and the levels of financial development display significant variations across countries and time. The savings rates range from less than 5% in Indonesia in the early 1960s to more than 50% in China today; the private credit to GDP ratios also range from less than 0.1 in the early years in those developing countries to larger than 2 in developed economies today.

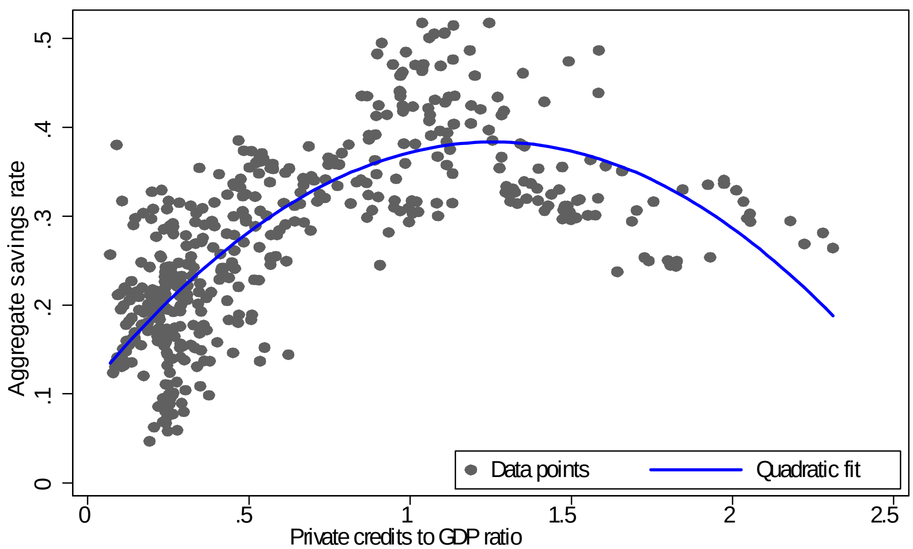

Figure 1. The aggregate savings rates and the private credit to GDP ratio, 12 Asian economies

Figure 1 displays the scatterplot of the aggregate savings rates against the private credit to GDP ratios for the 12 Asian economies, stacked with the quadratic fit line. It is evident from the figure that financial development and the aggregate savings rates display a hump-shaped relationship, with savings rates initially increasing and then decreasing with financial development. This pattern is consistent with what Horioka and Terada-Hagiwara (2012) have found. It is our main purpose to confirm this hump-shaped relationship using a broader sample and more robust econometric methods in the remaining parts of this section.

2.2 Specification

Consider the following reduced-form regression equation:

$$s_{it} = \kappa_1 s_{it-1} + \kappa_2' X_{it} + \alpha_i + u_{it}, \tag{1}$$

where s denotes the aggregate savings rate, X is a vector of explanatory variables reflecting financial development, age structure, rates of return, uncertainty, fiscal policy, income level and growth, which have all been shown to affect the savings rate in the literature, α denotes the unobserved country fixed effects, and u_{it} is the error term. Note that equation (1) nests both the dynamic specification with $\kappa_1 \neq 0$ and the static specification with $\kappa_1 = 0$. We consider both of them in this paper.

Various theories have emphasized different factors in explaining savings behavior. Life-cycle models shows that demographic factors play a nontrivial role in determining the savings rate, so each economy's aged dependency ratio, youth dependency ratio and life expectancy feature in the regression. Precautionary savings theory predicts that people will save against future uncertainties, thus the inflation rate is employed to capture macroeconomic uncertainty and public expenditures on health and education is used to reflect the uncertainty about future health and education expenditures. The permanent income hypothesis says that income and its growth determine economic agents' consumption and savings, so per-capita real GDP and its square, and the growth rate of per capita real GDP all play a part in the regressions. The quadratic term of income is included in the model in an attempt to capture the potential nonlinear relationship between income and the savings rate. Both Park and Shin (2009) and Horioka and Terada-Hagiwara (2012) find that the relationship is convex in their samples of Asian economies. Several other common variables, such as real interest rate and current account balance, are also included as in the literature.

However, the main purpose of our regression is to investigate the presumably nonlinear effects of financial development on the savings rate, after properly controlling for those relevant factors mentioned above. There are various measures of financial development in the literature, some of which focus on the size of the whole financial sector, while others focus on the role of financial intermediation.[7] We use the private credit to GDP ratio as our main measure of financial development, following King and Levine (1993), Loayza, Schmidt-Hebbel and Serven (2000), and Horioka and Terada-Hagiwara (2010) among many others.[8] Nevertheless, for the purpose of robustness check, we also consider four different measures of financial development that are popular in the literature. The four measures are: deposit money bank assets to GDP ratio, stock market capitalization to GDP ratio, M2 to GDP ratio and financial market depth measured by the sum of outstanding domestic private debt securities and stock market capitalization to GDP ratio. In order to capture the possible nonlinearity, we employ both the level and its quadratic term as explanatory variables.

[7] See the survey by Cook (2003) and the survey by Schmidt-Hebbel and Serven (2002). Thorsten Beck also writes about "two concepts of financial development", with one being the "financial intermediation view", and the other being the "financial center view". For details, please refer to his article at http://www.voxeu.com/index.php?q=node/7185

[8] This measure reflects the "financial intermediation" view of financial development in Thorsten Beck's article. To be sure, a larger financial sector does not necessarily imply a higher level of financial development. As an example, Thorsten Beck points to Nigeria in the 1980s, where the expansion of the financial sector was accompanied by "financial dis-intermediation". Another example is present day China, where large-scale state-owned banks provide limited credits to private firms. On the contrary, aggregate credits provided to the private sectors measure directly the activities of financial intermediation. Hence, we consider it as a more appropriate measure of financial development *in the context of this paper*.

2.3 Results

We use the standard within estimators and random effects estimators to estimate the static models. Following Loayza, Schmidt-Hebbel and Serven (2000), we use the Arellano-Bond GMM-IV estimator to estimate the dynamic models. In the estimation, we treat the two dependency ratios, life expectancy and public expenditures, as strictly exogenous variables, and assume that all the others are weakly exogenous.

In deriving the results using dynamic models, we instrument the endogenous variables use their first two feasible lags in the first-differenced equation and using the first lag difference in the level equation. We also employ the financial reform index prepared by Abiad, Detragiache and Tressel (2008) as an external instrument, as Roodman (2009a) finds that lags of several popular financial development measures, including private credit used in this paper, all perform badly in the system GMM estimation when he replicates the exercises conducted by Levine, Loayza and Beck (2000). To deal with the problem of "too many instruments", we also consider reducing the instrument count in our regression exercises by "collapsing" the instrument matrix, as suggested and exemplified by Roodman (2009a, 2009b). In Appendix B, we discuss the econometric methodology we use in more detail.

The coefficients of most interest in model (1) are those on financial development and its quadratic term. Table 1 summarizes the regression coefficients on these two variables for our full sample of 41 economies, using three different estimators. Tables C.1 – C.3 in Appendix C report the full results.[9] We briefly discuss these results now.

All the regressions show that the relationship between financial development and the aggregate savings rate is hump-shaped. The regressions confirm our observations in Figure 1, and are consistent with the results presented by Horioka and Terada-Hagiwara (2012), who only considered the 12 Asian economies. Indeed, the last two columns of Table C.1 in Appendix C replicate Horioka and Terada-Hagiwara's results.

The regression results for other explanatory variables are broadly consistent with those of Loayza, Schmidt-Hebbel and Serven (2000) and Horioka and Terada-Hagiwara (2012). For example, the two age dependencies show significant negative effects on the savings rate, as senior citizens tend to consume their previous savings and chridern usually consume without their own income. Per capita GDP growth has a highly significant positive effect in almost all cases. Real interest and inflation rates have ambiguous effects. Higher public expenditure typically has a negative effect, which may reflect households' precautionary saving incentives, to some extent. The current account surplus has a positive and significant effect.

[9] The results are robust if we don't collapse the instrument matrix, however, the Hansen tests perform extremely bad, as discussed by Roodman (2009a,b). So we only report the results when the instrument matrix is collapsed. We do not report the coefficients and statistics on life expectancy, current account balance, and public expenditures either in Tables C.1 – C.3 to save the space. The full results are available upon request.

Table 1. Summary of the coefficients on private credit and its quadratic term (full sample)

	(1)	(2)	(3)
Estimator	GMM-IV	Within	RE
Collapsed IV	Yes	–	–
Private credit	0.0321**	0.0925***	0.105***
	(0.0144)	(0.0287)	(0.0308)
Private credit sq.	-0.0127***	-0.0296***	-0.0321***
	(0.00491)	(0.0100)	(0.0101)
Country effects	Yes	Yes	No
Observations (No. of cn)	809 (37)	976 (41)	976 (41)

2.4 Robustness

To further check the robustness of the hump-shaped relationship, we consider the aforementioned four additional measures of financial development. For simplicity, we use only the within estimator and random-effects estimator. Tables C.4 – C.5 in Appendix C report the results. All of the results confirm a significant hump-shaped relationship between financial development and the aggregate savings rate.

While quadratic fit is enough to capture nonlinearity, it is also interesting to see what happens under semiparametric regression, into which financial development enters non-parametrically and other explanatory variables enter parametrically. In Appendix D, we apply the Baltagi and Li's (2002) estimator for partially linear fixed-effects panel data models to our full sample, and find that the relationship is indeed hump-shaped.

Thus, we confirm what Horioka and Terada-Hagiwara (2012) find in their sample of 12 developing Asian economies. In fact, a hump-shaped relationship is not surprising if both households and firms face financial constraints, as our model will show.

3 The Model

We consider an infinite-horizon economy. There is no aggregate uncertainty. The economy has two types of agents, households and firms, with equal mass normalized to unity. Firms accumulate capital and combine labor and capital to produce consumption goods. Households trade bonds, supply labor to the firms, and own their shares. Both households and firms are subject to uninsurable idiosyncratic risks and financial constraints specified below.

3.1 Households

The household side is similar to that modelled by Wen (2009, and 2015). Households have idiosyncratic preference shock, θ_t, which is assumed to be i.i.d across individuals and over time with cumulative distribution function $F(\theta_t)$. The idiosyncratic state of households is (s_t, a_t, θ_t), where s_t is the savings in bonds, and a_t is the share of stocks. So as not to risk confusion, we do not explicitly express decision variables as functions of aggregate states. The timing is as follows. Each time period is divided into two sub-periods. The idiosyncratic shocks are realized in the second sub-period. The household with (s_t, a_t) chooses labor supply n_t in the first sub-period without observing her own θ_t, and chooses consumption c_t and saving in bonds s_{t+1} and a share of stocks a_{t+1} in the second sub-period after observing θ_t. With such an information and market structure, labor income cannot be used to fully diversify the idiosyncratic risk, and savings become a buffer stock to smooth consumption. Taking as given the real interest rate R_{bt} and real wages W_t, a household with state (a_t, s_t, θ_t) chooses optimal $n_t(a_t, s_t)$, $c_t(a_t, s_t, \theta_t)$, $s_{t+1}(a_t, s_t, \theta_t)$ and $a_{t+1}(a_t, s_t, \theta_t)$ to solve the following recursive problem:

$$V_t(a_t, s_t, \theta_t) = \max_{\{c_t, a_{t+1}, s_{t+1}\}} \left[\max_{n_t} (\theta_t \log c_t - \psi n_t) \right] + \beta \int V_{t+1}(a_{t+1}, s_{t+1}, \theta_{t+1}) \, dF(\theta_{t+1}), \tag{2}$$

where $\beta \in (0,1)$ is the discount rate. The budget constraint faced by the household is

$$c_t + s_{t+1}/R_{bt} + a_{t+1}Q_t \leq s_t + W_t n_t + (Q_t + D_t) a_t. \tag{3}$$

where Q_t is the stock price and D_t is the dividend. Notice that the term $s_{t+1}/R_{bt} + a_{t+1}Q_t$ indicates a household's total savings (or borrowing). Households face financial frictions. We assume that each household is subject to a limited borrowing capacity:

$$s_{t+1}/R_{bt} + a_{t+1}Q_t \geq -B_t, \tag{4}$$

where the borrowing limit B_t is, for simplicity, assumed to be exogenous for the household. Appendix F.1 derives the households' optimal decision rules.

3.2 Firms

A typical firm combines labor $\tilde{n}_t$ and capital k_t to produce output y_t through Cobb-Douglas technology, $y_t = k_t^{\alpha} \tilde{n}_t^{1-\alpha}$, where $\alpha \in (0,1)$. Following Kiyotaki and Moore (2012), we assume that each firm encounters an idiosyncratic investment efficiency shock, ε_t, which is assumed to be i.i.d. across firms and over time with cumulative distribution function (CDF) $G(\varepsilon_t)$. The idiosyncratic state of a firm is thus (k_t, ε_t). The capital accumulation of each firm follows

$$k_{t+1} = (1-\delta) k_t + \varepsilon_t i_t, \tag{5}$$

where i_t is the investment made in period t.

As with heterogeneous households, the firm's dynamic optimization problem becomes slightly more complicated. The key is to find the right discounting factor. Following the asset pricing literature,[10] we

[10] See, for example, Hansen and Richard (1987), Cochrane (1991) and among others.

assume that a pricing kernel $\rho\frac{\Lambda_{t+1}}{\Lambda_t}$ ($\rho < 1$) exists, and the firm's optimization problem is given by[11]

$$J_t(k_t, \varepsilon_t) = \max_{\{\tilde{n}_t, k_{t+1}, i_t\}} k_t^{\alpha} \tilde{n}_t^{1-\alpha} - W_t \tilde{n}_t - i_t + \rho \frac{\Lambda_{t+1}}{\Lambda_t} \int J_{t+1}(k_{t+1}, \varepsilon_{t+1})\, dG(\varepsilon_{t+1}), \tag{6}$$

subject to capital accumulation rule (5), liquidity constraint and collateral constraint (to be specified later).

As the optimal labor choice is static, it is straightforward to show that the firm's operating profits are linear in capital stock. Specifically, the profit is $R_t k_t$, where $R_t = \alpha \left(\frac{1-\alpha}{W_t}\right)^{\frac{1-\alpha}{\alpha}}$ is the rate of return to capital. The dividend, d_t, is hence $R_t k_t - i_t$, and the optimal labor demand for an individual firm is

$$\tilde{n}_t(k_t) = \left(\frac{1-\alpha}{W_t}\right)^{\frac{1}{\alpha}} k_t. \tag{7}$$

The firm uses both internal funds $R_t k_t$ and external funds l_t to finance the investment expenditure. The maximum investment is thus subject to the liquidity constraint

$$i_t \leq l_t + R_t k_t. \tag{8}$$

For simplicity, assume that the firm raises the external funds through intra-period loans. Specifically, the firm borrows from financial intermediaries at the beginning of period t and pays them back with zero interest at the end of period t, by raising additional equity.[12] As in Kiyotaki and Moore (1997), loans are subject to collateral constraints. The firm pledges a fraction $\xi \in (0, 1]$ of its own fixed assets k_t at the beginning of period t as collateral. At the end of period t, the expected market value of the collateral is equal to $\rho\frac{\Lambda_{t+1}}{\Lambda_t} \int J_{t+1}(\xi k_t, \varepsilon_{t+1})\, dG(\varepsilon_{t+1})$, which is the discounted expected market value of the firm if it owns capital stock ξk_t at the beginning of period $t+1$ and faces the same investment and collateral constraints in the future. The amount of loans l_t cannot exceed this collateral value, otherwise the firm would choose to default on its debt. This leads to the following collateral constraint:

$$l_t \leq \rho \frac{\Lambda_{t+1}}{\Lambda_t} \int J_{t+1}(\xi k_t, \varepsilon_{t+1})\, dG(\varepsilon_{t+1}). \tag{9}$$

To sum up, each firm chooses $\tilde{n}_t(k_t)$, $i_t(k_t, \varepsilon_t)$, $k_{t+1}(k_t, \varepsilon_t)$ and $l_t(k_t, \varepsilon_t)$ to solve the problem (6) subject to constraints (5), (8) and (9). Appendix F.2 derives the firms' optimal decision rules.

3.3 Financial Intermediary

The financial intermediary holds a portfolio consisting of stocks of all firms and collects aggregate dividends D_t from them, i.e., $D_t = \int d_t(k_t, \varepsilon_t)\, d\kappa(k_t, \varepsilon_t)$, where $\kappa(k_t, \varepsilon_t)$ is the joint distribution of k_t and

[11] Here ρ is a discount factor, but notice that because of heterogeneity on the household side, ρ does not necessarily equal the households' discount factor β.

[12] Thus, dividends may be negative at the firm level.

ε_t. The price of the portfolio Q_t is thus given by

$$Q_t = \rho \frac{\Lambda_{t+1}}{\Lambda_t} \left(Q_{t+1} + D_{t+1} \right). \tag{10}$$

The financial intermediary is introduced for the sole purpose of simplifying the notation of the households' maximization problem. One can instead assume that the households directly hold a market portfolio consisting of stocks in all firms, and the equilibrium results will be the same.

3.4 General Equilibrium

Let $\chi(a_t, s_t, \theta_t)$ be the joint distribution of the idiosyncratic state of households (a_t, s_t, θ_t). Let $X_t = \int x_t(k_t, \varepsilon_t)\, d\kappa(k_t, \varepsilon_t)$ be the aggregate counterpart of x_t on the firm side, where $x_t = \{k_{t+1}, i_t, \tilde{n}_t, y_t, d_t\}$, and let $Z_t = \int z_t(a_t, s_t, \theta_t)\, d\chi(a_t, s_t, \theta_t)$ be the aggregate counterpart of z_t on the household side, where $z_t = \{n_t, s_{t+1}, c_t\}$. The general equilibrium is then defined as sequences of aggregate variables $\{X_t, Z_t\}$, individual firms' choices $\{k_{t+1}(k_t, \varepsilon_t), i_t(k_t, \varepsilon_t), \tilde{n}_t(k_t, \varepsilon_t), y_t(k_t, \varepsilon_t), d_t(k_t, \varepsilon_t)\}$, individual households' choices $\{n_t(a_t, s_t), s_{t+1}(a_t, s_t, \theta_t), c_t(a_t, s_t, \theta_t)\}$ and prices $\{Q_t, W_t, R_t, R_{bt}\}$, such that each firm and each household solve their individual optimization problems, and all markets clear. In particular, the labor market clears; the bond market clears: $S_{t+1} = 0$; the stock market clears: $\int a_{t+1}(a_t, s_t, \theta_t)\, d\chi(a_t, s_t, \theta_t) = 1$; and the good market clears: $Y_t = C_t + I_t$. In addition, aggregate capital accumulates according to $K_{t+1} = (1-\delta) K_t + \int \varepsilon_t i_t(k_t, \varepsilon_t)\, d\kappa(k_t, \varepsilon_t)$. Appendix F.3 derives the full dynamic system of the aggregate economy.

4 The Savings Rate and Financial Development

In this section, we discuss the relationship between the aggregate savings rate and the financial development in the stationary equilibrium. According to the aggregate resource constraint, the aggregate savings rate, s, is measured by the investment-to-output ratio, i.e., $s = I/Y$ (or $1 - C/Y$). The parameter ξ measures financial development on the firm side. For households, we assume that the exogenous borrowing limit B_t in (4) is linear in Q_t, i.e., $B_t = bQ_t$.[13] The parameter b thus measures financial development on the household side. To better understand the hump-shaped relationship between financial development and the savings rate, we consider four different scenarios: (i) the economy without any financial constraints (the first-best allocation), (ii) the economy with financial constraints on firms only, (iii) the economy with financial constraints on households only, and (iv) the economy with both firms and households subject to financial constraints.

[13] This specification guarantees the existence of a balance growth path. Our results are quite robust to the functional form of B_t.

4.1 The First-Best Allocation

The first-best allocation is identical to that in a standard RBC model. Consider an economy without any financial frictions. The two borrowing constraints (4) and (9) then never bind. It is easy to show that each individual's consumption is proportional to the idiosyncratic shock θ_t,[14] and the stationary equilibrium interest rate $R_b = 1/\beta$ ($\mathbf{KS^{NF}}$ curve in Figure 2). As the households are the capital suppliers, constant interest rate implies that the capital supply scheme (R_b, s) is a horizontal line. Meanwhile, as the firms' borrowing constraints do not bind, the marginal q equals $1/\varepsilon_{\max}$, where $\varepsilon_{\max}$ is the upper bound of investment efficiency ε. As a result, interest rate equals gross marginal product of capital (MPK) in the stationary equilibrium, i.e.,

$$R_b = \varepsilon_{\max}\alpha Y/K + (1-\delta). \tag{11}$$

Since the savings rate s in this case is $\frac{\delta K/\varepsilon_{\max}}{Y}$, the above equation gives the capital demand scheme ($\mathbf{KD^{NF}}$ curve in Figure 2). It also implies that the equilibrium savings rate is $s^{\mathbf{FB}} = \frac{\alpha\delta\beta}{1-(1-\delta)\beta}$, which is identical to that in a standard RBC model. The intersection A in Figure 2 illustrates this case.

4.2 Financial Constraints only on Firms

Assume that only firms are subject to financial constraints. The bonds holding decision remains the same as that in the first-best case, which implies that $R_b = 1/\beta$. Meanwhile, it can be shown (see Appendix F.4) that the capital demand is defined by an implicit function $\mathbf{KD}(s, R_b, \xi) = 0$ ($\mathbf{KD^F}$ curve in Figure 2). As higher ξ induces financially constrained firms to demand more capital, an increase in ξ shifts the demand curve upward. This is why the capital demand curve $\mathbf{KD^F}$ in this case is to the left of that in the first-best case ($\mathbf{KD^{NF}}$). The equilibrium savings rate is then at the intersection (point B in Figure 2) of $\mathbf{KD^F}$ and $\mathbf{KS^F}$. The following proposition states the relationship between the equilibrium savings rate and financial development.

Proposition 1 *If only firms are subject to financial constraints, then the savings rate in the stationary equilibrium increases with the financial development level.*

The above proposition is intuitive. Borrowing constraints on firms prevent them from undertaking investments that yield a positive net present value, a problem commonly referred to as the *underinvestment* problem. To see this, recall that in the first-best case (where ξ is sufficiently large), the MPK equals the interest rate. In this case, the interest rate remains at a constant $1/\beta$, while the MPK is relatively high, because capital demand is relatively low due to the financial constraints. As a result, the equilibrium MPK (social return of capital) is higher than the interest rate. The wedge between the social return of capital and the interest rate prevents households from investing in capital to its socially optimal level,

[14] The first order conditions for consumption, labor and bond holding imply that $\theta_t/c_t(a_t, s_t, \theta_t)$ is identical across households.

even though capital yields high social return. In the stationary equilibrium, the households need to pay q for one unit of capital (indirectly through buying stocks), while the social cost of building one unit of capital is just one. In other words, households pay a premium to acquire capital, which weakens their incentives to save. An increase in ξ leads to a decrease in q, which increases the households' incentives to save.

4.3 Financial Constraints only on Households

When only households are subject to the financial constraints, the marginal q is equal to $1/\varepsilon_{\max}$, and as in equation (11) we have $R_b = MPK = \alpha\delta/s + (1-\delta)$, which gives the capital demand

$$s = \frac{\alpha\delta}{R_b - 1 + \delta}. \tag{12}$$

Therefore, in this case the capital demand curve is the same as that in the first-best case ($\mathbf{KD^{NF}}$ in Figure 2). On the other hand, the optimal bond-holding decisions (see the discussions in Appendix F.3) lead to a positive relationship between savings and real interest rate from the suppliers of loanable funds, which implicitly defines the capital supply curve $\mathbf{KS}(R_b, s, b) = 0$ ($\mathbf{KS^F}$ in Figure 2). The equilibrium interest rate R_b and savings rate s are jointly determined by the above demand-supply scheme (intersection C in Figure 2). As an increase in financial development b induces households to demand more loans (or provide less saving), higher b shifts the capital supply curve upward, implying that the equilibrium savings rate s declines. The following proposition summarizes the result.

Proposition 2 *If only households are subject to financial constraints, then the savings rate in the stationary equilibrium decreases with the level of financial development.*

The above proposition is also intuitive. Borrowing constraints prevent households from fully insuring their idiosyncratic uncertainties (captured by θ_t), the precautionary motive makes the households tend to over-save compared to the first-best case. Consequently, the equilibrium interest rate R_b is less than the first-best level $1/\beta$ (see intersection C in Figure 2). The wedge between R_b and $1/\beta$ reflects the liquidity premium, which is the extra benefit for each unit of savings.

4.4 Financial Constraints on both Households and Firms

When both households and firms are subject to financial constraints, the aggregate equilibrium is characterized by a system of equations (see (F.19) to (F.26) in Appendix F.3). Appendix F.4 shows that the equilibrium interest rate R_b and savings rate s are implicitly determined by the following demand and supply functions (corresponding to $\mathbf{KD^F}$ and $\mathbf{KS^F}$ in Figure 2)

$$\mathbf{KD}(s, R_b; \xi) = 0, \tag{13}$$

$$\mathbf{KS}(s, R_b; b, \xi) = 0. \tag{14}$$

Intersection D in Figure 2 illustrates this case.

The financial development on the firm side, ξ, affects both capital demand and capital supply. An increase in ξ simultaneously shifts $\mathbf{KD^F}$ curve upward and $\mathbf{KS^F}$ curve downward. Intuitively, higher ξ induces higher demand for capital from firms due to the relaxing borrowing constraint. Meanwhile, higher ξ increases the rate of return on equity as the marginal q declines. Hence, an increase in ξ raises the savings rate, but the effect on the interest rate is ambiguous.

The financial development on the household side, b, only affects capital supply. In particular, higher b raises the households' borrowing capacity, inducing the financially constrained households to demand more loans. As a result, the total capital supply declines. Hence, an increase in b shifts the $\mathbf{KS^F}$ curve upward, resulting in a lower savings rate and a higher interest rate. The following proposition summarizes the results.

Proposition 3 *If both households and firms are subject to financial constraints, then the savings rate in the stationary equilibrium increases with the financial development* ξ *but decreases with the financial development* b.

The above proposition also suggests that if financial development increases b and ξ simultaneously, the overall effect of financial development on the savings rate would be ambiguous. An increase in households' borrowings reduces the savings rate but an increase in firms' borrowing increases it. The following section illustrates how it is possible for this model to explain the observed hump-shaped relationship between financial development and the savings rate.

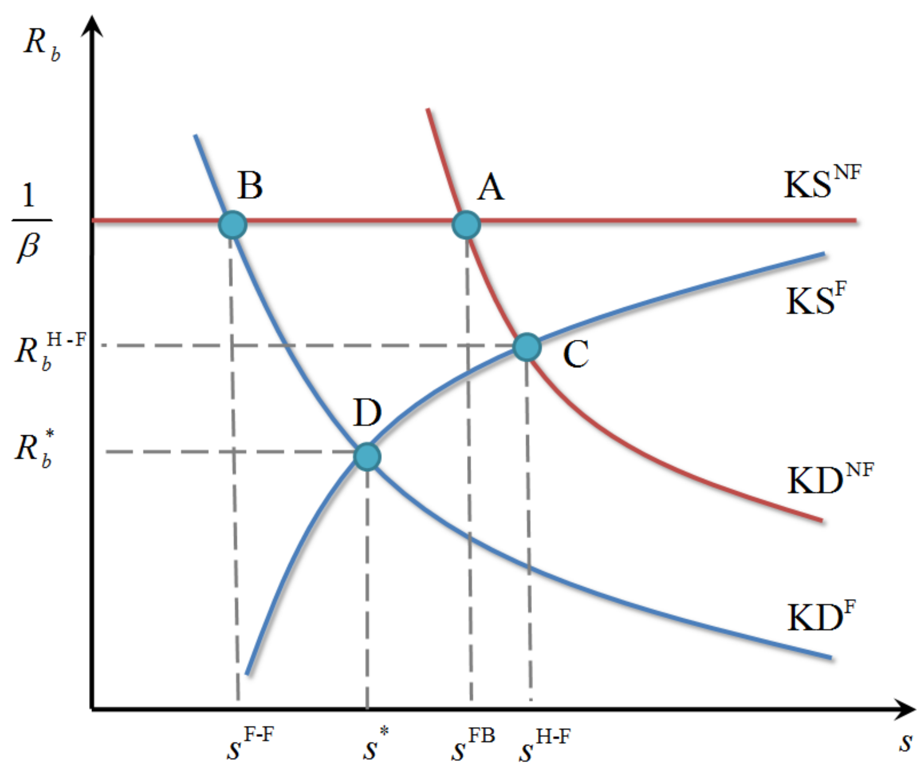

Figure 2. The equilibrium savings rates in different cases

Notes: The KS and KD are capital supply and demand curves, respectively. The superscripts "F" and "NF" stand for with frictions and without frictions, respectively. For instance, KS^{NF} is the capital supply without frictions on the household side and KD^{NF} is the capital demand without frictions on the firm side. The intersections A, B, C, D correspond to the equilibrium savings rates and interest rates in the following four cases: first-best, financial constraint only on the firm side, financial constraint only on the household side, and financial constraints on both the firm and household sides.

5 Explaining the Hump-Shaped Relationship: Quantitative Analysis

We now calibrate and simulate the model to explain the hump-shaped relationship we observe in the data. Since we only consider the stationary equilibrium of the model, here we exploit the cross-sectional data instead of the panel data. As in the previous empirical part, a sample of 12 Asian economies and 31 OECD economies is studied. For each economy, we calculate the average savings rate and the average private credits-to-GDP ratio (relative to that in U.S.) over 1995-2008. Then we generate the corresponding simulated cross-sectional data in the model.

The model is calibrated as follows. We partition the parameters into two sets. The first set, $\Theta_1 = \{\alpha, \beta, \delta\}$, are standard parameters, which, we assume, take common values across countries. The second set, Θ_2, contains the financial friction parameters and those pertaining to the distributions of idiosyncratic shocks, which are country specific. Assume that idiosyncratic investment efficiency shock ε and preference shock θ follow Log-normal distributions with means μ_ε and μ_θ and standard deviations σ_ε and σ_θ, then $\Theta_2 = \{b, \xi, \mu_\varepsilon, \sigma_\varepsilon, \mu_\theta, \sigma_\theta\}$.[15] For Θ_1, we follow the standard business cycle literature to set the discounting factor β to 0.985, the capital share α to 0.36 and the depreciation rate δ to 0.025. For parameters in set Θ_2, we calibrate their values by matching the model-implied moments to their counterparts in real data. We first calibrate the U.S. economy, then calibrate the other countries by taking the U.S. as a benchmark.

The first two parameters in Θ_2 pertain to borrowing limits, so we calibrate them based on the U.S. household and firm finance data. Specifically, b captures the financial tightness on the household side. In the model, b is defined as the ratio of household borrowing to the value of equity, $\frac{s_{it}+Q_t}{Q_t}$. It thus reflects the change in household debt relative to the change in the value of equity, i.e., $\frac{\Delta \text{Household Debt}}{\Delta \text{Household Equity}}$. According to Mian and Sufi (2011), U.S. households borrowed 25 cents on every dollar of additional home equity value in 1997, so we set b^{US} to 0.25. Similarly, the parameter that governs the firms' borrowing constraints, ξ, is defined as $\frac{L_{jt}}{Q_{jt}}$ (or $\frac{\Delta \text{Loan}}{\Delta \text{Equity}}$). According to Covas and den Haan (2011), the average ratio of the change in firms' liability to the change of firms' value of assets is 62%, so we set ξ^{US} to 0.62. Since μ_θ does not affect the steady-state great ratios, we normalize $\mu_\theta = 1$ for all countries. For the remaining three parameters $\{\mu_\varepsilon, \sigma_\varepsilon, \sigma_\theta\}$, we pin down their values by matching three aggregate moments in the

[15] Notice that our theorectical analysis shows that the results do not rely on the specifications of the distributions. Our quantitative results are robust for the well-behaved distributions.

U.S. economy: savings rate, capital-output ratio and real interest rate.[16] Table 2 reports the calibrated parameter values and the model-implied moments for the U.S. economy.

Table 2. Calibration of U.S. economy

Parameter values								
α	β	δ	b	ξ	μ_ε	σ_ε	μ_θ	σ_θ
0.36	0.985	0.025	0.25	0.62	0.056	0.315	1	3.875

Targeted moments			
	Savings rate	Capital/Output	Real interest rate
Data	0.16	2.23	0.05
Model	0.16	2.23	0.05

Now we calibrate the parameter set Θ_2 of the remaining countries. We assume that the distribution parameters of θ, $\{\mu_\theta, \sigma_\theta\}$, are the same as those in the U.S. For the distribution parameters of ε, $\{\mu_\varepsilon, \sigma_\varepsilon\}$, we set σ_ε to the same value in the U.S. and calibrate the mean μ_ε.[17] For b and ξ, as we do not have direct empirical evidence for each country, we pin them down by matching the empirical moments. As a result, Θ_2 is reduced to $\{b, \xi, \mu_\varepsilon\}$ for each country. The three empirical moments we choose as targets are savings rate, $\frac{\text{Domestic output}}{\text{U.S. output}}$ and $\frac{\text{Domestic private credits-to-GDP}}{\text{U.S. private credits-to-GDP}}$.[18]

[16] The parameter σ_θ reflects households' uncertainty about the consumption, It thus affects households' saving behavior, while the parameters μ_ε and σ_ε affect firms' investment decision and capital accumulation. Consequently, aggregate moments such as savings rate, and capital-output ratio as well as real interest rate, are informative to calibrate the values of $\{\sigma_\theta, \mu_\varepsilon, \sigma_\varepsilon\}$. The data details about the savings rate and the real interest rate are presented in Appendix A. The value of the capital-output ratio is taken from Greenwood et.al. (2013).

[17] This is equivalent to assuming that the investment efficiency ε is the sum of a country-specific investment-specific technology (IST) and an idiosyncratic shock with the same log-normal distribution across countries.

[18] All of these moments in each country are the average value over 1995-2008. Appendix A provides data details about these empirical moments.

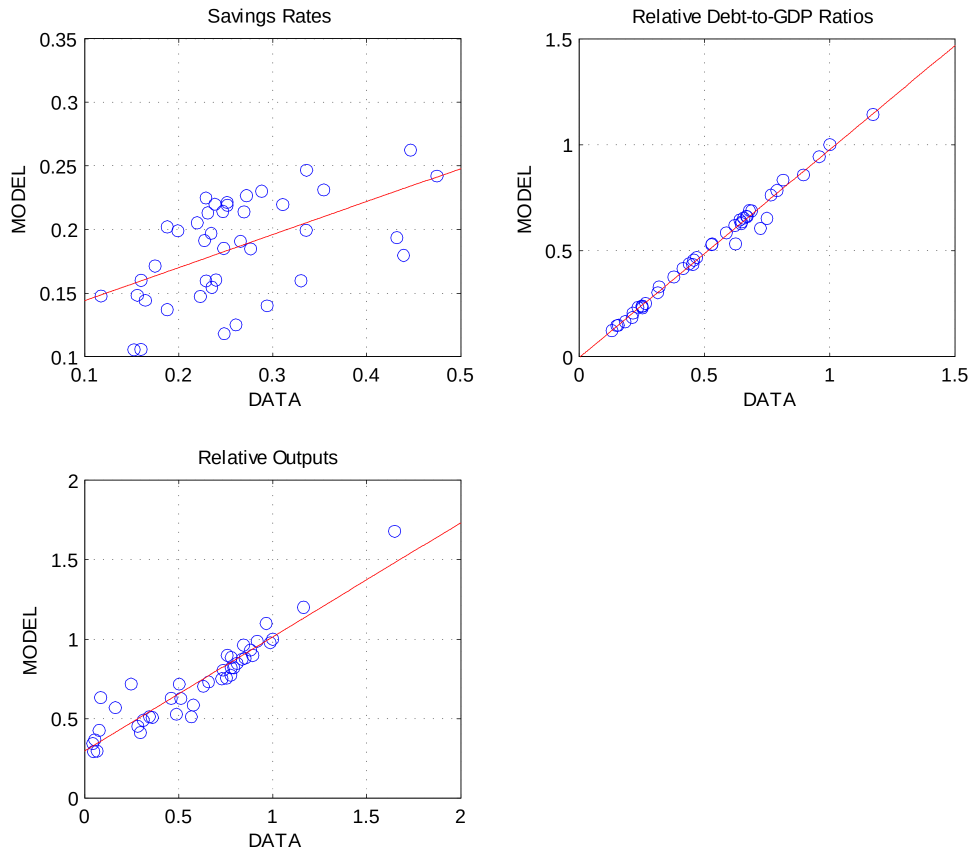

Figure 3. Savings rates, relative private credit-to-GDP ratios and relative outputs: data vs. model

To see how well our model can explain the variation of savings rates in the data, Figure 3 compares the model simulated variables and their counterparts in the data. The first panel shows that two sets of savings rates are significantly correlated, with the correlation around 0.54, indicating that our model can explain the observed variation of savings rates fairly well.[19] The remaining two panels show that our model indeed perform very well to explain the cross-country variations of credit-to-GDP ratios (relative to the U.S.) and of output (relative to the U.S.). The correlations bewteen the model simulated data and the real data for these two variables are 0.99 and 0.94, respectively.

We now discuss to what extent our model can explain the hump-shaped relationship. We simulate the savings rate and the private credit-to-GDP ratio (relative to the U.S.) for each country. Figure 4 plots the results. The right panel presents a significant unconditional hump-shaped relationship between the savings rate and financial development. It also shows that the simulated pattern is close to the one we observe in the real data (the left panel). To control the effects of other factors, we run the following

[19] In the alternative calibration strategy, we assume that the variance of θ, σ_θ^2, is country-specific. In this case four country-specific parameters $\{\sigma_\theta, \mu_\varepsilon, b, \xi\}$ are calibrated by targeting four empirical moments (three in the benchmark calibration and the average real interest rate). Based on this calibration, our model simulated savings rates can closely fit the data, and the result of hump-shaped relationship is robust.

regression for both the real and the model simulated data:[20]

$$\text{Savings rate} = \beta_0 + \beta_1 \times (\text{Private credits-to-GDP}) + \beta_2 \times (\text{Private credits-to-GDP})^2 + \beta_3 \times \text{Controls}. \quad (15)$$

Table 3 reports the results. It clearly indicates a significant hump-shaped relationship between the savings rate and the private credits-to-GDP ratio in the model simulated data (i.e., $\hat{\beta}_1 > 0$ and $\hat{\beta}_2 < 0$). A similar pattern can also be found in the real data, indicating that our model explains the data quite well.

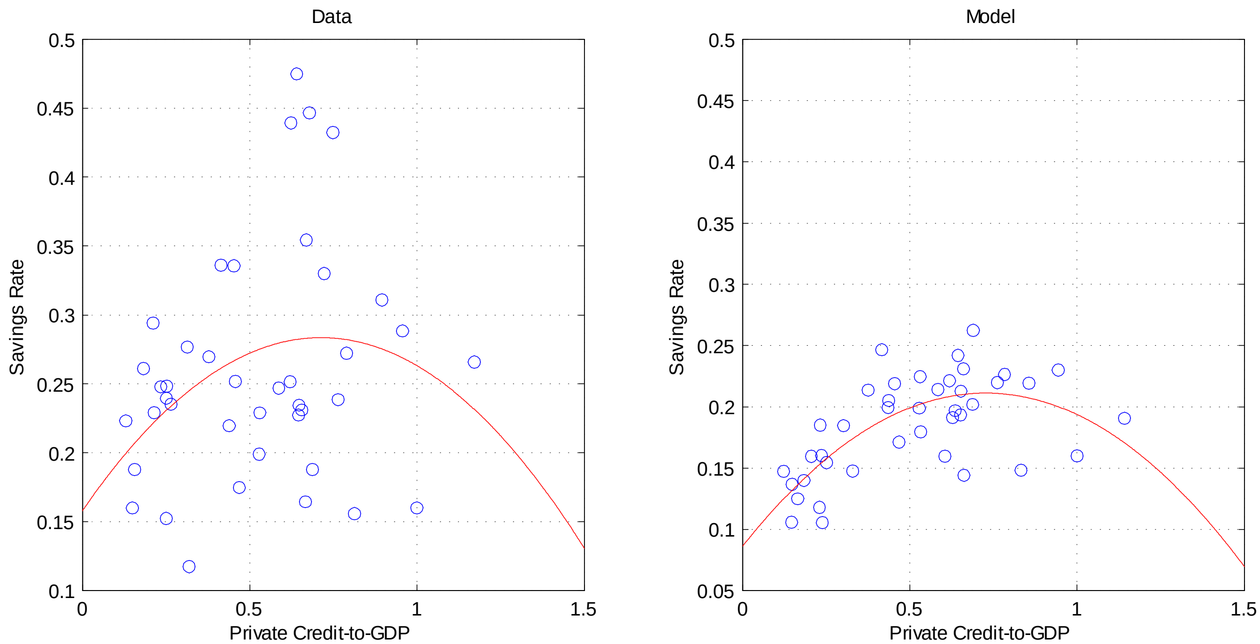

Figure 4. The unconditional hump-shaped relationship: data v.s. model

Table 3. Cross-country regressions

	Data	Model
β_1	0.3964	0.2162
s.e.	(0.1561)	(0.0529)
β_2	-0.3131	-0.1433
s.e.	(0.1173)	(0.0454)
Adjusted R^2	0.78	0.76

In the model, we show that the hump-shaped relationship stems from the prediction that the savings rate is decreasing in b and increasing in ξ. To confirm this, we use the model-simulated data, and regress the savings rate on parameters b and ξ, controlling for country-specific parameter μ_ε. The result shows that both the coefficients are significant under the 1% level and the signs are consistent with our prediction. In particular, the coefficient of b is -0.0162 with standard error 0.0040, and the coefficient of ξ is 0.0658 with standard error 0.0204.

[20] For the regression using real data, the controls are the same as those in the previous empirical part. For the regression using model simulated data, the control is the country-specific parameter μ_ε.

6 Conclusion

This study documents a hump-shaped relationship between financial development and the aggregate savings rate, and provides a model to explain the relationship. The model shows that financial development increases savings if only firms are financially constrained, but reduces savings if only households are borrowing constrained. When both firms and households are financially constrained, the model predicts a hump-shaped relationship between financial development and the aggregate savings rates as observed in the data. The model also shows that financial frictions can cause either undersaving or oversaving, depending on the stage of financial development. A natural policy recommendation would be that the optimal taxation on capital income may either be negative (in the case of undersaving) or positive (in the case of oversaving). Given its enormous welfare consequences, a more complete characterization of the optimal capital income taxation should be pursued in the future. With both firms and households being financially constrained, the model also generates another interesting prediction: the coexistence of high returns on capital and low real interest rates due to the saving wedge and investment wedge (see Gourinchas and Jeane (2013)), which is consistent with the evidence from emerging economies.[21] An interesting implication of this prediction is that if these economies were to open up to countries with more advanced financial markets, financial capital would flow out from the former to the latter, while physical capital would flow (in terms of FDI) in the opposite direction to enjoy high capital returns (see evidence by Ju and Wei (2011)). Wang, Wen and Xu (2014) explain such two-way flows in the spirit of this study.

References

[1] Abiad, A., E. Detragiache, and T. Tressel, 2008. A New Database of Financial Reforms, IMF Working Paper.

[2] Aiyagari, R., 1994. Uninsured Idiosyncratic Risk and Aggregate Saving, The Quarterly Journal of Economics, vol. 109(3), pages 659-84.

[3] Albuquerque, R., and H. Hopenhayn, 2004. Optimal Lending Contracts and Firm Dynamics, Review of Economic Studies, vol. 71(2), pages 285-315.

[4] Arellano, M., and S. Bond, 1991. Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations, Review of Economic Studies, vol. 58, pages 277-297.

[5] Arellano, M., and O. Bover, 1995. Another Look at the Instrumental Variable Estimation of Error-Component Models, Journal of Econometrics, vol. 68, pages 29-51.

[21] See Wen (2009) for an alternative model to explain this fact. Wen (2009) assumes an imperfectly competitive (state-owned) banking sector generating a spread between the deposit rate and the loan rate. Here we obtain the same effect by allowing financial constraints on the firm side.

[6] Baltagi, B. and D. Li, 2002. Series Estimation of Partially Linear Panel Data Models with Fixed Effects, Annals of Economics and Finance, vol. 3, pages 103-116.

[7] Bandiera, O., G. Caprio, Honohan and F. Schiantarelli, 2000. Does Financial Reform Raise or Reduce Saving?, The Review of Economics and Statistics, vol. 82(2), pages 239-263.

[8] Beck, T., 2009. The Econometrics of Finance and Growth, in Palgrave Handbook of Econometrics, vol. 2, Terence Mills and Kerry Patterson ed., 1180–1211. Houndsmill: Palgrave Macmillan.

[9] Beck, T., B. Buyukkarabacak, F. Rioja and N. Valev, 2012. Who Gets the Credit? And Does It Matter? Household vs. Firm Lending Across Countries, The B.E. Journal of Macroeconomics, Berkeley Electronic Press, vol. 12(1), pages 2.

[10] Bernanke, B., and M. Gertler, 1989. Agency Costs, Net Worth, and Business Fluctuations, American Economic Review, vol. 79(1), pages 14-31.

[11] Bernanke, B., M. Gertler and S. Gilchrist, 1999. The Financial Accelerator in a Quantitative Business Cycle Framework, Handbook of Macroeconomics, in: J. B. Taylor and M. Woodford (ed.), Handbook of Macroeconomics, edition 1, volume 1, chapter 21, pages 1341-1393 Elsevier.

[12] Blundell, R., and S. Bond, 1998. Initial Conditions and Moment Restrictions in Dynamic Panel Models, Journal of Econometrics, vol. 87(1), pages 115-143.

[13] Carroll, C., D., 1992. The Buffer-Stock Theory of Saving: Some Macroeconomic Evidence, Brookings Papers on Economic Activity, Economic Studies Program, The Brookings Institution, vol. 23(2), pages 61-156.

[14] Chen, K., A. Imrohoroglu and S. Imrohoroglu, 2006. The Japanese Saving Rate, American Economic Review, Vol. 96(5), 1850-1858.

[15] Chen, K., A. Imrohoroglu and S. Imrohoroglu, 2007. The Japanese Saving Rate between 1960-2004: Productivity or Demographics, Economic Theory, Vol. 32, 87-104.

[16] Cochrane, J., 1991. Production-Based Asset Pricing and the Link between Stock Returns and Economic Fluctuations, Journal of Finance, vol. 46(1), pages 209-37.

[17] Cook, C., 2003. Does Financial Depth Improve Aggregate Savings Performance? Further Cross-Country Evidence, Review of Development Economics, vol. 7(2), pages 248–265

[18] Cooley, T., R. Marimon and V. Quadrini, 2004. Aggregate Consequences of Limited Contract Enforceability, Journal of Political Economy, vol. 112(4), pages 817-847.

[19] Covas, F., and W.J. Den Haan, 2011. The Cyclical Behavior of Debt and Equity Finance. The American Economic Review, pages 877-899.

[20] Gale, D. and M. Hellwig, 1985. Incentive-Compatible Debt Contracts: The One-Period Problem, Review of Economic Studies, vol. 52(4), pages 647-63.

[21] Gourinchas, P.O., and O. Jeanne, 2013. Capital Flows to Developing Countries: The Allocation Puzzle. The Review of Economic Studies, pages, 1-32.

[22] Gross, D., and S. Souleles, 2002. An empirical analysis of personal bankruptcy and delinquency. Review of Financial Studies, vol 15(1), pages, 319-347.

[23] Hansen, L. and S. Richard, 1987. The Role of Conditioning Information in Deducing Testable, Econometrica, vol. 55(3), pages 587-613.

[24] Holmstrom, B., and J. Tirole, 1997. Financial Intermediation, Loanable Funds, and the Real Sector, The Quarterly Journal of Economics, vol. 112(3), pages 663-91.

[25] Horioka, C., and A. Terada-Hagiwara. 2010. The Determinants and Long-term Projections of saving rates in Developing Asia. ADB Economics Working Paper Series No. 228, Asian Development Bank, Manila.

[26] Horioka, C., and T. Yin. 2010. A Panel Analysis of the Determinants of Household Saving in the OECD Countries: The Substitutability of Social Safety Nets and Credit Availability. Institute of Social and Economic Research, Osaka University.

[27] Hubbard, G., 1998. Capital-Market Imperfections and Investment, Journal of Economic Literature, American Economic Association, vol. 36(1), pages 193-225, March.

[28] Inessa L., 2003. Financial Development and Financing Constraints: International Evidence from the Structural Investment Model, Review of Financial Studies, vol. 16(3), pages 765-791.

[29] Ju, J., and S.J. Wei, Domestic Institutions and the Bypass Effect of International Capital Flow, American Economic Journal: Economic Policy, November 2010, pages 173-204 .

[30] Kimball, M., 1990. Precautionary Saving in the Small and in the Large, Econometrica, vol. 58(1), pp 53-73.

[31] King, R., and R. Levine, 1993. Finance, Entrepreneurship and Growth: Theory and Evidence," Journal of Monetary Economics, vol. 32(3), pages 513-542.

[32] Kiyotaki, N. and J. Moore, 1997. Credit Cycles, Journal of Political Economy, vol. 105(2), pages 211-48.

[33] Kiyotaki, N., ahd J. Moore, 2012. Liquidity, business cycles, and monetary policy. National Bureau of Economic Research Working Paper (No. w17934).

[34] Levine, R., N. Loayza and T. Beck, 2000. Financial Intermediation and Growth: Causality and Causes, Journal of Monetary Economics vol. 46, pages 31-77.

[35] Loayza, N., K. Schmidt-Hebbel and L. Serven, 2000. What Drives Private Saving across the World? The Review of Economics and Statistics, Vol. 82(2), pages 165-181.

[36] Mian, A., and A. Sufi, 2011. House Prices, Home Equity-Based Borrowing, and the US Household Leverage Crisis. American Economic Review, vol 101(5), pages 2132-2156.

[37] Park, D., and K. Shin. 2009. Saving, Investment, and Current Account Surplus in Developing Asia. ADB Economics Working Paper Series No. 158, Economics and Research Department, Asian Development Bank, Manila.

[38] Roodman, D., 2009a. A Note on the Theme of Too Many Instruments, Oxford Bulletin of Economics and Statistics, vol. 71(1), pages 135-158.

[39] Roodman, D., 2009b. How to Do xtabond2: an Introduction to Difference and System GMM in Stata, The Stata Journal, vol. 9(1), pages 86-136.

[40] Stiglitz, J., and A. Weiss, 1981. Credit Rationing in Markets with Imperfect Information, American Economic Review, vol. 71(3), pages 393-410.

[41] Townsend, R., 1979. Optimal Contracts and Competitive Markets with Costly State Verification, Journal of Economic Theory, vol. 21(2), pages 265-293.

[42] Wang, P., and Y. Wen, 2012. Hayashi meets Kiyotaki and Moore: A Theory of Capital Adjustment Costs, Review of Economic Dynamics, vol. 15, pages 207-225.

[43] Wang, P., Y. Wen and Z. Xu, 2014. Two-Way Capital Flows and Global Imbalances, Working paper, HKUST.

[44] Wen, Y, 2009. Saving and Growth under Borrowing Constraints: Explaining the High Savings Rate Puzzle, Federal Reserve Bank of St. Louis Working Paper 2009-045C.

[45] Wen, Y, 2015. Money, liquidity and welfare, European Economic Review, vol 76, pages 1-25.

Appendix

A Sample Description

Tables A.1-A.3 describe the samples used in the paper. In Table A.3, OECD countries with an asterisk are not included in the household saving regressions to be described later, due to unavailability of data. Table A.4 describes the summary statistics of the variables used in the aggregate savings rate regressions. Table A.5 describes the summary statistics of different financial development indices, including the financial reform index prepared by Abiad, Detragiache and Tressel (2008). Table A.6 presents the correlation matrix of these financial development indices. Table A.7 describes the summary statistics of the variables used in the OECD household savings rate regressions.

Table A.1: Variables used in the aggregate savings rate regression

Variable	**Data source**	**Note**
Gross domestic savings rate	WDI	% of GDP
Domestic credits to private sector	WDI	% of GDP
Aged dependency ratio	WDI	Pop. aged 65 and above/total pop.
Youth dependency ratio	WDI	Pop. aged 0-14/total pop.
Consumer price inflation	WDI	Annual rate
Real interest rate	WDI	Annual rate
Per capita GDP	WDI	In constant 2000 US dollar
Per capita GDP growth	WDI	Annual rate
Social expenditure	PWT 7.0	% of GDP
Current account balance	WDI	% of GDP, positive if surplus
Life expectancy	WDI	Life expectancy at birth, in years
Financial reform index	Abiad et al. (2008)	Covering seven aspects of financial sector policies

Table A.2: Variables used in the household savings regression

Variable	**Data source**	**Note**
Household savings rate	OECD	% of household disposable income
Household credits	OECD	Household Loans/household disposable income
Aged dependency ratio	WDI	Pop. aged 65 and above/total population
Youth dependency ratio	WDI	Pop. aged 0-14/total population
Consumer price inflation	WDI	Annual rate
Real interest rate	WDI	Annual rate
P/c household disp. income growth	OECD	Annual rate
P/c household disp. income	OECD	In constant 2005 US dollar
Social expenditures	OECD	% of GDP
Life expectancy	WDI	Life expectancy at birth, in years
Domestic credits to private sector	WDI	% of GDP
Financial reform index	Abiad et al. (2008)	Covering seven aspects of financial sector policies

Table A.3: Economies in the sample

Country	Code	Region/Group	Country	Code	Region/Group
Australia	AUS	OECD	Norway	NOR	OECD
Belgium	BEL	OECD	Poland	POL	OECD
Canada	CAN	OECD	Portugal	PRT	OECD
Czech Rep.	CZE	OECD	Slovak Rep.	SVK	OECD
Denmark	DNK	OECD	Slovenia	SVN	OECD
Estonia	EST	OECD	Spain	ESP	OECD
Finland	FIN	OECD	Sweden	SWE	OECD
France	FRA	OECD	Switzerland	CHE	OECD
Germany	DEU	OECD	United Kingdom	GBR	OECD
Greece	GRC	OECD	United States	USA	OECD
Hungary	HUN	OECD	China	CHN	Asia
Iceland *	ISL	OECD	Hong Kong	HKG	Asia
Ireland *	IRL	OECD	India	IND	Asia
Israel *	ISR	OECD	Indonesia	IDN	Asia
Italy	ITA	OECD	Malaysia	MYS	Asia
Japan *	JPN	OECD/Asia	Pakistan	PAK	Asia
Korea	KOR	OECD/Asia	Philippines	PHL	Asia
Luxembourg	LUX	OECD	Singapore	SGP	Asia
Mexico *	MEX	OECD	Thailand	THA	Asia
Netherlands	NLD	OECD	Vietnam	VNM	Asia
New Zealand	NZL	OECD			

Table A.4: Summary statistics of the variables used in the aggregate savings rate regressions

Variable	Mean	Std. Dev.	Min.	Max.	N
Savings rate	0.249	0.081	0.017	0.532	1505
Private credits/GDP	0.712	0.462	0.07	3.195	1397
Inflation rate	0.067	0.070	-0.096	0.584	1544
Real interest rate	0.039	0.044	-0.246	0.352	1241
Per capita GDP growth	0.032	0.032	-0.143	0.187	1580
Per capita GDP (log)	8.852	1.455	4.807	10.94	1582
Balance of current account	-0.006	0.054	-0.265	0.328	1328
Social expenditures	0.087	0.039	0.011	0.237	1405
Aged dependency ratio	0.148	0.075	0.021	0.304	1602
Youth dependency ratio	0.345	0.101	0.126	0.898	1602
Life expectancy (log)	4.274	0.108	3.717	4.414	1576

Table A.5: Summary statistics of different financial development indices

Variable	Mean	Std. Dev.	Min.	Max.	N
Private credits/GDP	0.712	0.462	0.07	3.195	1397
Deposit money bank assets/GDP	0.701	0.446	0.086	2.7	1475
Market capitalization/GDP	1.093	0.811	0.051	5.95	592
Stock market capitalization/GDP	0.695	0.67	0.002	5.805	750
M2/GDP	0.695	0.436	0.08	2.996	1289
Financial reform index (normalized)	0.624	0.299	0.000	1.000	1047

Table A.6: Correlation matrix of different financial development indices

	pc	basset	mktcap	stockcap	m2	finref
Private credits/GDP	1					
Deposit money bank assets/GDP	0.893***	1				
Market capitalization/GDP	0.649***	0.514***	1			
Stock market capitalization/GDP	0.552***	0.495***	0.914***	1		
M2/GDP	0.807***	0.883***	0.514***	0.595***	1	
Financial reform index	0.499***	0.460***	0.444***	0.374***	0.408***	1

* $p<0.05$, ** $p<0.01$, *** $p<0.001$

Table A.7: Summary statistics of the variables used in OECD household savings rate regressions

Variable	Mean	Std. Dev.	Min.	Max.	N
Household savings rate	0.055	0.056	-0.131	0.232	363
Household credits	0.731	0.436	0.001	1.987	294
Per capita household disposable income growth	0.023	0.026	-0.072	0.135	321
Per capita household disposable income (log)	9.532	0.388	8.202	10.375	363
Inflation rate	0.039	0.048	-0.023	0.379	406
Real interest rate	0.043	0.034	-0.1	0.156	332
Social expenditures	0.088	0.03	0.036	0.194	406
Aged dependency ratio	0.212	0.044	0.08	0.304	406
Youth dependency ratio	0.276	0.061	0.198	0.602	406
Life expectancy (log)	4.348	0.035	4.213	4.407	406

B Empirical Methodology

As decribed in the main text, besides the standard static panel data regressions with fixed effects or random effects, we also employ the dynamic panel data regression with fixed effects, because Wooldridge test for first-order serial correlation of the errors always rejects the null hypothesis of zero autocorrelation in our static panel regressions, which implies that the time series display serious inertia.

Introducing lagged dependent variable in the regression, however, causes endogeneity problem. It is also highly possible that credit, income and its growth are jointly determined with the savings rate, so they might be correlated with the errors as well. As a result, the standard within estimator and IV estimator might produce inconsistent estimations. Specifically, to eliminate the unobserved country-specific effects in estimation, we need to work with the mean-difference or first-differenced model derived from equation (1). However, neither the within estimator nor IV estimation using lags is feasible in the mean-difference model, because any lag s_{it} is correlated with $\bar{u}_i$ and hence $(u_{it} - \bar{u}_i)$, which is the error term in the mean-difference model. The OLS estimator is also inconsistent in the first-differenced model, because Δs_{it-1} is correlated with Δu_{it}, which is the error term now.

Loayza, Schmidt-Hebbel and Serven (2000) suggest using the GMM-IV estimators developed by Arellano and Bond (1991), Arellano and Bover (1995) and Blundell and Bond (1998) to deal with the inertia and the endogeneity problem. Arellano and Bond (1991) work with the first-differenced model. They describe the later-called Arellano-Bond difference estimator. The estimator assumes that the errors are serially uncorrelated and $E(s_{iT}\Delta u_{it}) = 0$ for $T \leq t-2$. With this assumption, $\{s_{it-j}\}_{j=2,3,...}$can be used as instruments in the first-differenced model, and 2SLS/GMM estimation using lags as instruments delivers consistent and efficient results. They also develop a test for the serial correlation of the first-differenced errors.

However, lags could be weak instruments in the first-differenced model if the corresponding level variables display serious inertia over time. Arellano and Bover (1995) and Blundell and Bond (1998) propose the later-called system estimator, which employs both the level equation (1) and its first difference, to deal with this problem. They assume $E(\Delta s_{it-1}u_{it}) = 0$ in addition, so s_{it} can be instrumented using its own first lag difference Δs_{it-1} in the level equation. The system estimator is shown to be be more precise and to have better finite sample properties, and it can also be applied to control for the endogeneity of the other explanatory variables. Thus, we focus on this estimator in the following regressions.

In the regressions, we follow Loayza, Schmidt-Hebbel and Serven (2000) and assume that the endogenous explanatory variables are "weakly exogenous". Namely, $E(X_{it}u_{iT}) \neq 0$ for $T \leq t$, but $E(X_{it}u_{iT}) = 0$ for $T > t$, since the past realization of explanatory variables is less likely to be influenced by future innovations to the savings rate. With this assumption, weakly exogenous variables in X_{it} can be instrumented using their own lags $\{X_{it-j}\}_{j=2,3,...}$ in the first-differenced model. In the regressions, we treat the two dependency ratios, life expectancy and public expenditures as strictly exogenous variables, and assume that all the others are weakly exogenous.

The main results are presented in Section 2.3 and Appendix C. In deriving the results, we instrument the endogenous variables using their first two feasible lags in the first-differenced equation and using the first lag difference in the level equation. We also employ the financial reform index prepared by Abiad, Detragiache and Tressel (2008) as an external instrument, as Roodman (2009a) finds that lags of several popular financial development measures, including private credit used in this paper, all perform badly in the system GMM estimation when he replicates the exercises conducted by Levine, Loayza and Beck (2000).

It is also worth mentioning that using lags as instruments typically generates numerous instruments. Roodman (2009a) shows that "too many instruments" can overfit the endogenous variables and weaken the Hansen test for joint validity of instruments, making the results unreliable. Thus, we also consider reducing the instrument count in our regression exercises by "collapsing" the instrument matrix, as suggested and exemplified by Roodman (2009a, 2009b).

C Aggregate Savings Rate Regression

This appendix presents the main results of aggregate savings rate regressions. We use three different estimators: within estimator and random-effects estimator that apply to the static models, and the system estimator that applies to the dynamic model. We also consider three different samples: the sample of Asian economies studied by Horioka and Terada-Hagiwara (2010), the sample of 31 OECD economies, and finally the full sample consisting of both the Asian and OECD economies. Tables C.1 – C.3 present the results on the three different samples.

In all of the following tables, we report the estimated coefficients, their standard errors and the significance levels. The robust standard errors are displayed in the brackets under the coefficients. However, please notice that we don't report the coefficients and statistics on life expectancy, current account balance and public expenditures to save the space in Table C.1 – C.3. The coefficients on these three variables are broadly consistent with those in the literature. The full results are available upon request.

We also report several tests: the Wald test for the joint significance of the coefficients, the Hansen test for the validity of the instruments, Wooldridge test for the first-order autocorrelation of the residuals in the static model and the Arellano-Bond test for the autocorrelations of the first difference of the residuals in the dynamic model. Arrelano-Bond estimator assumes that the errors are serially uncorrelated, which implies Δu_{it} is first-order serially correlated, but not second-or-higher-order serially correlated. The Arrelano-Bond tests in all of the following tables support this assumption.

There exist two tests for over-identification of the system, namely the Sargen test and the Hansen test. the Sargan test is not weakened by the problem of "too many instruments", however, it is not robust. On the other hand, the Hansen test is robust, but it might be weakened by many instruments. Since we alrealdy solve the problem of "too many instruments" by collapsing the instrument matrix, we choose the robust Hansen test as our reference. The results of tests in all of the following tables show that our choice of instruments is valid.

The main results are robust if we use three lags as intruments in the first-differenced equation, while the results are mixed if only one lag is used. Specifically, when only one lag is used, in all cases the coefficients still imply a concave relationship, but they are significant only in the East Asian subsample, no matter whether the IV matrix is "collapsed" or not. In the full sample, the coefficients are significant only when the IV matrix is not "collapsed". Nevertheless, as aforementioned, the Hansen tests for joint validity of instruments in Tables C.1 – C.3 indicate that using two lags as instruments is valid. Thus, we interpret those results as being supportive of a hump-shaped relationship between financial development and the savings rate. Our semiparametric regression in Appendix D further confirms this.

To further check the robustness of the hump-shaped relationship, we also consider four additional measures of financial development in Tables C.4 – C.5 by running fixed-effect and random-effect regressions. The four measures are: deposit money bank assets to GDP ratio, financial market depth, stock market capitalization to GDP ratio and M2 to GDP ratio, where we measure the financial market depth by the sum of outstanding domestic private debt securities and stock market capitalization to GDP ratio. The tables show that there does exist a significant hump-shaped relationship between the aggregate savings rate and financial development, regardless of which measure we employ.

Table C.1: Aggregate savings rate regression: dynamic panel, Asia subsample

	(1)	(2)	(3)
Estimator	GMM-System	Within	RE
Number of IV	32	–	–
Collapsed IV	Yes	–	–
Lagged saving rate	0.805***	–	–
	(0.0948)		
Private credit	0.109**	0.140***	0.261***
	(0.0531)	(0.0206)	(0.0263)
Private credit sq.	-0.0271*	-0.0318***	-0.0816***
	(0.0160)	(0.00817)	(0.0115)
Inflation rate	-0.210**	0.0696*	0.134**
	(0.102)	(0.0420)	(0.0555)
Real interest rate	-0.213*	0.0257	0.0306
	(0.127)	(0.0544)	(0.0682)
Per capita GDP growth	0.209	0.272***	0.673***
	(0.129)	(0.0546)	(0.0765)
Per capita GDP (log)	-0.0205	0.0986***	0.183***
	(0.0645)	(0.0328)	(0.0333)
Per capita GDP (log) sq.	0.000999	-0.000233	-0.0109***
	(0.00402)	(0.00235)	(0.00207)
Aged dependency ratio	-0.281	-1.490***	-0.470***
	(0.233)	(0.158)	(0.157)
Youth dependency ratio	0.169	-0.0317	-0.172**
	(0.111)	(0.101)	(0.0724)
Wald test (p-val)	0.0000	0.0000	0.0000
Hansen test (p-val)	1.000	–	–
Wooldridge test (p-val)	–	0.0002	0.0002
Arellano-Bond test (p-val):			
1st-order autocorr.	0.009	–	–
2nd-order autocorr.	0.615	–	–
3rd-order autocorr.	0.320	–	–
Observations (No. of countries)	265 (12)	313 (12)	313 (12)

Notes: * p<0.1, ** p<0.05, *** p<0.01, and is the same for all of the following tables. In deriving the results, we instrument the endogenous variables using their first two feasible lags in the first-differenced equation and using the first lag difference in the level equation. We also employ the financial reform index prepared by Abiad, Detragiache and Tressel (2008) as an external instrument in the level equation. We also consider reducing the instrument count in our regression exercises by "collapsing" the instrument matrix, as suggested and exemplified by Roodman (2009a, 2009b). The same is true for Table C.2 and C.3.

Table C.2: Aggregate savings rate regression: dynamic panel, OECD subsample

	(1)	(2)	(3)
Estimator	GMM-System	Within	RE
Number of IV	32	–	–
Collapsed IV	Yes	–	–
Lagged saving rate	0.830***	–	–
	(0.0485)		
Private credits	0.0241	0.0552**	0.0513**
	(0.0188)	(0.0225)	(0.0230)
Private credits sq.	-0.0132**	-0.0168**	-0.0165*
	(0.00530)	(0.00802)	(0.00848)
Inflation rate	0.00758	0.0304	0.0239
	(0.0440)	(0.0456)	(0.0465)
Real interest rate	-0.0853	-0.136***	-0.169***
	(0.0568)	(0.0485)	(0.0516)
Per capita GDP growth	0.279***	0.200***	0.235***
	(0.0763)	(0.0610)	(0.0578)
Per capita GDP (log)	-0.0779	0.329*	0.259
	(0.0874)	(0.188)	(0.176)
Per capita GDP (log) sq.	0.00413	-0.0136	-0.0107
	(0.00460)	(0.0102)	(0.00996)
Aged dependency ratio	-0.0945***	-0.612***	-0.583***
	(0.0318)	(0.195)	(0.190)
Youth dependency ratio	-0.0155	-0.278**	-0.316***
	(0.0259)	(0.124)	(0.0996)
Wald test (p-val)	0.0000	0.0000	0.0000
Hansen test (p-val)	0.492	–	–
Wooldridge test (p-val)	–	0.0000	0.0000
Arellano-Bond test (p-val):			
1st-order autocorr.	0.001	–	–
2nd-order autocorr.	0.316	–	–
3rd-order autocorr.	0.114	–	–
Observations (No. of countries)	603 (27)	731 (31)	731 (31)

Table C.3: Aggregate savings rate regression: dynamic panel, full sample

	(1)	(2)	(3)
Estimator	GMM-System	Within	RE
Number of IV	32	–	–
Collapsed IV	Yes	–	–
Lagged saving rate	0.846***	–	–
	(0.0514)		
Private credit	0.0321**	0.0925***	0.105***
	(0.0144)	(0.0287)	(0.0308)
Private credit sq.	-0.0127***	-0.0296***	-0.0321***
	(0.00491)	(0.0100)	(0.0101)
Inflation rate	0.00472	0.0332	0.0403
	(0.0613)	(0.0424)	(0.0431)
Real interest rate	-0.0662	-0.0644	-0.0866
	(0.0844)	(0.0510)	(0.0553)
Per capita GDP growth	0.288***	0.245***	0.291***
	(0.101)	(0.0495)	(0.0484)
Per capita GDP (log)	-0.0146	0.174**	0.180***
	(0.0561)	(0.0780)	(0.0658)
Per capita GDP (log) sq.	0.000835	-0.00582	-0.00779*
	(0.00338)	(0.00480)	(0.00404)
Aged dependency ratio	-0.0744	-0.969***	-0.861***
	(0.0453)	(0.246)	(0.203)
Youth dependency ratio	-0.00880	-0.110	-0.212**
	(0.0219)	(0.131)	(0.0944)
Wald test (p-val)	0.0000	0.0000	0.0000
Hansen test (p-val)	0.549	–	–
Wooldridge test (p-val)	–	0.0000	0.0000
Arellano-Bond test (p-val):			
1st-order autocorr.	0.002	–	–
2nd-order autocorr.	0.452	–	–
3rd-order autocorr.	0.397	–	–
Observations (No. of countries)	809 (37)	976 (41)	976 (41)

Table C.4: Aggregate savings rate reg: alternative measures, within estimator, full sample

	(1)	(2)	(3)	(4)
Estimator	Within	Within	Within	Within
Financial development proxy	Bank assets	Mkt depth	Stock mkt cap	M2
Financial development	0.0439***	0.0210***	0.0109*	0.0385***
	(0.0118)	(0.00620)	(0.00567)	(0.0133)
Financial development sq.	-0.0133***	-0.00357***	-0.00254**	-0.0102**
	(0.00442)	(0.00108)	(0.00113)	(0.00457)
Inflation rate	0.0319*	0.0752**	0.0348	0.0482**
	(0.0182)	(0.0292)	(0.0251)	(0.0190)
Real interest rate	-0.0659***	-0.103***	-0.0338	-0.00942
	(0.0244)	(0.0319)	(0.0310)	(0.0257)
Per capita GDP growth	0.211***	0.136***	0.144***	0.182***
	(0.0345)	(0.0396)	(0.0385)	(0.0351)
Per capita GDP (log)	0.185***	0.192***	0.131***	0.171***
	(0.0218)	(0.0364)	(0.0351)	(0.0214)
Per capita GDP (log) sq.	-0.00537***	-0.00992***	-0.00387*	-0.00505***
	(0.00136)	(0.00236)	(0.00214)	(0.00134)
Current account balance	0.191***	0.147***	0.154***	0.199***
	(0.0221)	(0.0238)	(0.0230)	(0.0225)
Social expenditures	-0.892***	-0.756***	-1.198***	-0.979***
	(0.114)	(0.193)	(0.165)	(0.124)
Aged dependency ratio	-1.000***	-0.853***	-0.551***	-1.100***
	(0.0701)	(0.103)	(0.0992)	(0.0749)
Youth dependency ratio	-0.0831**	0.0125	0.00527	-0.128***
	(0.0419)	(0.0789)	(0.0746)	(0.0477)
Life expenditure	-0.265***	0.216*	-0.340***	-0.219***
	(0.0493)	(0.124)	(0.0906)	(0.0505)
Obs. (No. of countries)	966 (41)	519 (36)	636 (41)	874 (40)
Wald test (p-val)	0.000	0.000	0.000	0.000

Notes: "Bank assets" denotes deposit money bank assets to GDP ratio, "Mkt depth" denotes financial market depth, which is measured by the sum of outstanding domestic private debt securities and stock market capitalization to GDP ratio, "Stock mkt cap" denotes stock market capitalization to GDP ratio, and "M2" denotes M2 to GDP ratio. The same is true for Table C.5.

Table C.5: Aggregate savings rate reg: alternative measures, RE estimator, full sample

	(1)	(2)	(3)	(4)
Estimator	RE	RE	RE	RE
Financial development proxy	Bank assets	Mkt depth	Stock mkt cap	M2
Financial development	0.0590**	0.0234**	0.0176**	0.0557**
	(0.0301)	(0.00916)	(0.00723)	(0.0258)
Financial development sq.	-0.0166**	-0.00410***	-0.00374***	-0.0165**
	(0.00807)	(0.00155)	(0.00117)	(0.00710)
Inflation rate	0.0391	0.0768	0.0390	0.0544
	(0.0482)	(0.0866)	(0.0606)	(0.0490)
Real interest rate	-0.1000	-0.104**	-0.0489	-0.0363
	(0.0621)	(0.0480)	(0.0495)	(0.0635)
Per capita GDP growth	0.268***	0.169*	0.185**	0.241***
	(0.0594)	(0.0869)	(0.0790)	(0.0540)
Per capita GDP (log)	0.179***	0.154**	0.107	0.170***
	(0.0648)	(0.0715)	(0.0895)	(0.0648)
Per capita GDP (log) sq.	-0.00727*	-0.00730	-0.00368	-0.00686*
	(0.00404)	(0.00456)	(0.00547)	(0.00390)
Current account balance	0.228***	0.165***	0.173***	0.230***
	(0.0406)	(0.0474)	(0.0459)	(0.0436)
Social expenditures	-0.708***	-0.461	-0.789**	-0.658***
	(0.225)	(0.335)	(0.316)	(0.224)
Aged dependency ratio	-0.897***	-0.804***	-0.595***	-0.949***
	(0.224)	(0.190)	(0.184)	(0.227)
Youth dependency ratio	-0.234**	-0.0827	-0.173	-0.289***
	(0.0946)	(0.147)	(0.110)	(0.0926)
Life expectancy	-0.234	0.0379	-0.339	-0.213
	(0.259)	(0.279)	(0.228)	(0.284)
Obs. (No. of countries)	966 (41)	519 (36)	636 (41)	874 (40)
Wald test (p-val)	0.000	0.000	0.000	0.000

D Semiparametric Estimation

In this section, we apply the Baltagi-Li (2002) estimator for partially linear panel data models to our full sample. We choose this estimator among many others, simply because there exists a handy user-written Stata command `xtsemipar` to implement it. In the following, we draw from Baltagi and Li (2002) in introducing their methods.

Consider the following semiparametric specification:

$$s_{it} = \kappa' \tilde{X}_{it} + g(z_{it}) + \tilde{\alpha}_i + \tilde{u}_{it}, \tag{D.1}$$

where $\tilde{X}_{it}$ contains those explanatory variables of the savings rate explained in Section 2.2, except the measure of financial development (private credit), z_{it} denotes the ratio of aggregate private credit to GDP ratio, $\tilde{\alpha}_i$ is the unobserved country fixed effects, and finally $\tilde{u}_{it}$ is the error term.

Thus, financial development enters nonparametrically while the other explanatory variables enter parametrically in this partially linear specification. To eliminate the fixed effects, we choose to work with the first-differenced model:

$$\Delta s_{it} = \kappa' \Delta \tilde{X}_{it} + [g(z_{it}) - g(z_{it-1})] + \Delta \tilde{u}_{it}, \tag{D.2}$$

where Δ denotes the first difference. Baltigi and Li (2002) propose using series $p^K(z)$ of dimension $K \times 1$ to approximate $g(z)$, where $p^K(z)$ denotes the first K elements of series $\{p_j(z)\}_{j=1,2,3,\ldots}$. The series of functions have the property that as K grows there exists a linear combination of $p^K(z)$ that can approximate any $g(z)$ belonging to an additive class of functions arbitrarily well in mean square error. As a result, $g(z_{it}) - g(z_{it-1})$ can be approximated using $p^K(z_{it}) - p^K(z_{it-1})$ and equation (D.2) becomes:

$$\Delta s_{it} = \kappa' \Delta \tilde{X}_{it} + \gamma \left[p^K(z_{it}) - p^K(z_{it-1}) \right] + \Delta \tilde{u}_{it}. \tag{D.3}$$

Baltagi and Li (2002) show that κ and γ can be consistently estimated. Once κ and γ are known, we can get the residuals and hence by equation (D.1), we have

$$residuals = s_{it} - \kappa' \tilde{X}_{it} - \tilde{\alpha}_i = g(z_{it}) + \tilde{u}_{it}. \tag{D.4}$$

Thus, $g(z_{it})$ can be fitted by some standard univariate nonparametric regression of the above residuals on z_{it}. Table D.1 reports the coefficients on the other explanatory variables. It is evident that these results are consistent with what we obtained in Section 2.2, using parametric regression. Figure E.1 displays the nonparametric fit of the savings rate on private credit, using quartic B-spline smoothing. The graph looks similar if we set the power to 3, 5 or 6. It confirms that the relationship between the gross saving rate and financial development is hump-shaped.

Table D.1: Aggregate savings rate regression: Baltagi-Li estimator, full sample

Inflation rate	-0.0245
	(0.0381)
Real interest rate	-0.0978***
	(0.0351)
Per capita GDP growth	0.100***
	(0.0297)
Per capita GDP (log)	0.182
	(0.109)
Per capita GDP (log) sq.	-0.00201
	(0.00606)
Current account surplus	0.305***
	(0.0349)
Public expenditures	-1.110***
	(0.307)
Aged dependency ratio	-0.761***
	(0.231)
Youth dependency ratio	0.125
	(0.121)
Life expectancy	-0.292*
	(0.149)
Observations	920
R-squared	0.379

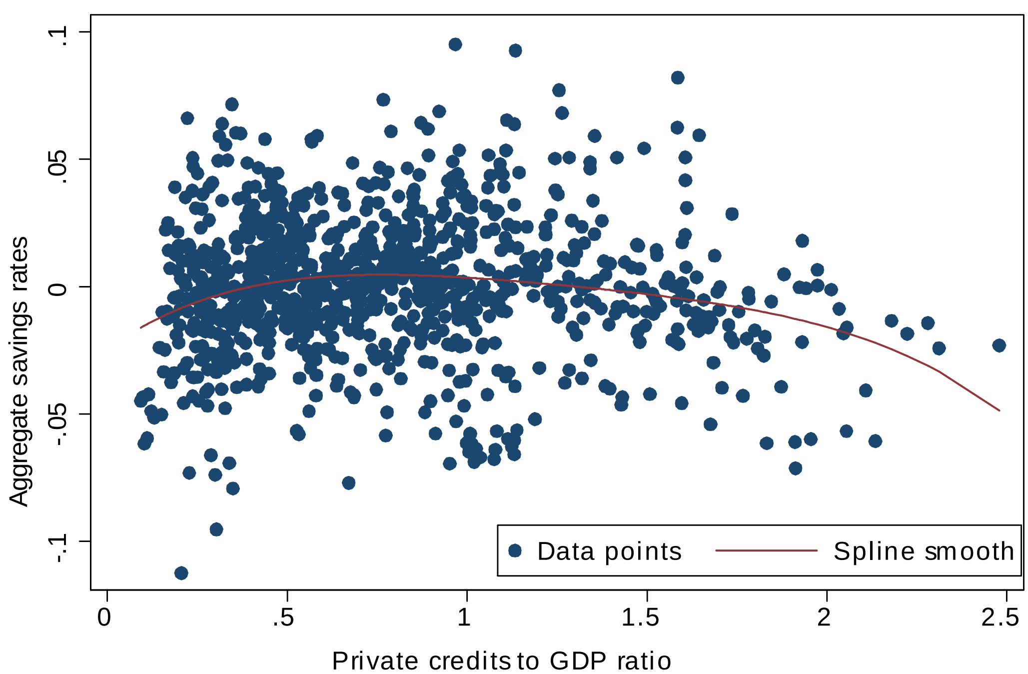

Figure E.1 Nonparametric fit of the aggregate savings rate on private credits to GDP ratio, full sample

E Household Savings Rate Regression

In this appendix, we investigate how the household savings rate changes with household credit for the sample of OECD economies. We focus on OECD economies, simply because of data accessibility. For Asian economies, there does not exist a single data source from which we could retrieve all the data needed in this regression.

We collect the annual time series data of the household savings rate, household credits, household disposable income and social expenditures for OECD economies from the OECD databases, whenever the data are available. The other potential explanatory variables are retrieved from World Development Indicators. As a result, we have a sample of 26 economies, with the time series ranging from 1995 to 2008, because the earliest observations on household finance begin in 1995. A detailed description of the sample is presented in Tables A.2, A.3 and A.7 of Appendix A.

Again, consider the following reduced-form regression equation:

$$s_{it}^h = \gamma_1 s_{it-1}^h + \gamma_2' X_{it}^h + \alpha_i^h + u_{it}^h, \tag{E.1}$$

where s^h denotes the household savings rate, X^h is a vector of explanatory variables reflecting the volume of household credit, household disposable income and its growth, age structure, social expenditure, the rates of return and uncertainty, all of which are important to household saving behavior, α^h denotes the country-specific effects, and finally u^h is the error term.

For the same reasons as those given in the main text for the aggregate savings rate regression, we choose to work with the static as well as the dynamic model. We use the ratio of household loans to household disposable income to measure the size of credits going to households.

Tables E.1 – E.3 summarize the results using different estimators and different specifications. We use three estimators: the Arellano-Bond system estimator for the dynamic specification in equation (E.1), the within estimator, and the RE estimator for static specification without the lagged dependent variable. Notice that in all of the three tables, we don't report the results on life expectancy to save the space. Full results are available upon request.

Table E.1 investigates if there exists a hump-shaped relationship between the household savings rate and household credit. To this end, we introduce the quadratic term. In column (1) and (2), we also control for the overall financial development using two different indices: the financial reform index prepared by Abiad, Detragiache and Tressel (2008) in column (1) and private credits to GDP ratio in column (2). The table shows that there does not exist any hump-shaped relationship between the household savings rate and the household credits. Instead, the household savings rate is monotonically decreasing in household credits.

Tables E.2 and E.3 confirm the monotonic relationship using different specifications and estimators. In Table E.2, we employ the dynamic panel data regression without controlling for the overall financial development, while in Table E.3 we control for the overall financial development using the same two different indices: the financial reform index in column (1) and (3), the private credit to GDP ratio in column (2) and (4). In column (1) of Tables E.2 and E.3, we instrument the endogenous variables using the first 2 lags in the first-differenced model. In column (2) of Table E.2 and E.3, we use the first 3 lags.

It is evident from the tables that the household savings rate is indeed decreasing in the volume of household credits. This result implies that households tend to save less when the level of household side financial development is high, which may justify our modeling of the household side in this paper.

Table E.1: OECD household savings reg: with quadratic terms

	(1)	(2)	(3)	(4)
Estimator	Within	Within	Within	RE
Household credits	-0.155**	-0.0731	-0.0664*	-0.0678*
	(0.0732)	(0.0603)	(0.0374)	(0.0351)
Household credits sq.	0.0331	0.00395	-0.00328	-0.00102
	(0.0275)	(0.0226)	(0.0149)	(0.0149)
Financial developent	-0.114	-0.00810	–	–
	(0.0734)	(0.0114)		
P-c household disp. inc. growth	0.298***	0.296***	0.287***	0.271***
	(0.0514)	(0.0599)	(0.0517)	(0.0569)
P-c household disp. inc. (log)	-0.135	-0.309	-0.337	-0.603**
	(0.777)	(0.704)	(0.336)	(0.307)
P-c household disp. inc. (log) sq.	0.0167	0.0244	0.0252	0.0369**
	(0.0418)	(0.0382)	(0.0181)	(0.0164)
Inflation rate	0.218	0.292*	0.287***	0.328***
	(0.154)	(0.142)	(0.0701)	(0.0769)
Real interest rate	0.137	0.188*	0.166***	0.225***
	(0.0878)	(0.0923)	(0.0626)	(0.0685)
Social expenditures	1.661	1.510*	1.494***	0.707***
	(0.983)	(0.819)	(0.345)	(0.235)
Aged dependency ratio	-0.449	-0.445	-0.381	-0.347*
	(0.398)	(0.348)	(0.247)	(0.183)
Youth dependency ratio	0.456	0.497**	0.424**	0.265*
	(0.267)	(0.184)	(0.191)	(0.141)
Wald test (p-val)	0.000	0.000	0.000	0.000
Observations (No. of countries)	194 (24)	224 (26)	229 (26)	229 (26)

Notes: This table investigates whether there exists a hump-shaped relationship between the household savings rate and household credits. In column (1) and (2), we control for the overall financial development using two different indices: the financial reform index prepared by Abiad, Detragiache and Tressel (2008) in column (1) and private credits to GDP ratio in column (2).

Table E.2: OECD household savings reg: without overall financial development

	(1)	(2)	(3)	(4)
Estimator	GMM-System	GMM-System	Within	RE
Number of IV	19	26	–	–
Collapsed IV	Yes	Yes	–	–
Lagged household saving rate	0.724***	0.791***	–	–
	(0.109)	(0.0598)		
Household credits	-0.0262**	-0.0255*	-0.0747***	-0.0705***
	(0.0107)	(0.0138)	(0.0183)	(0.0160)
P-c household disp. inc. (log)	-0.422**	-0.496**	-0.532	-0.709
	(0.215)	(0.231)	(0.694)	(0.534)
P-c household disp. inc. (log) sq.	0.0224**	0.0265**	0.0353	0.0422
	(0.0101)	(0.0109)	(0.0374)	(0.0300)
P-c household disp. inc. growth	-0.184	0.0711	0.283***	0.269***
	(0.284)	(0.123)	(0.0653)	(0.0597)
Inflation rate	0.171	0.233***	0.259**	0.332***
	(0.128)	(0.0795)	(0.119)	(0.0979)
Real interest rate	0.272***	0.328***	0.289**	0.393***
	(0.0844)	(0.0630)	(0.124)	(0.128)
Social expenditures	-0.107	-0.0633	1.366	0.618*
	(0.158)	(0.141)	(0.875)	(0.369)
Aged dependency ratio	0.0234	-0.0349	-0.296	-0.296
	(0.131)	(0.112)	(0.320)	(0.250)
Youth dependency ratio	-0.119	-0.0889	0.425*	0.271
	(0.134)	(0.131)	(0.214)	(0.195)
Wald test (p-val)	0.0000	0.0000	0.0000	0.0000
Hansen test (p-val)	0.567	0.560	–	–
Wooldridge test (p-val)	–	–	0.0000	0.0000
Arellano-Bond test (p-val):				
1st-order autocorr.	0.035	0.023	–	–
2nd-order autocorr.	0.255	0.313	–	–
3rd-order autocorr.	0.516	0.394	–	–
Observations (No. of countries)	170 (24)	170 (24)	229 (26)	229 (26)

Notes: In column (1), we instrument the endogenous variables using the first 2 lags in the first-differenced model. In column (2), we use the first 3 lags.

Table E.3: OECD household savings reg: with overall financial development

	(1)	(2)	(3)	(4)
Estimator	GMM-System	GMM-System	Within	Within
Number of IV	28	30	–	–
Collapsed IV	Yes	Yes	–	–
Lagged household saving rate	0.527***	0.805***	–	–
	(0.112)	(0.0893)		
Household credits	-0.0242*	-0.0274**	-0.0826***	-0.0658***
	(0.0144)	(0.0136)	(0.0201)	(0.0190)
Financial developent	-0.0390	0.00423	-0.130	-0.00656
	(0.0366)	(0.0201)	(0.0763)	(0.0112)
P-c household disp. inc. growth	-0.0557	0.235*	0.294***	0.294***
	(0.125)	(0.128)	(0.0504)	(0.0676)
P-c household disp. inc. (log)	0.345	0.341	-0.305	-0.500
	(0.334)	(0.444)	(0.709)	(0.674)
P-c household disp. inc. (log) sq.	-0.0195	-0.0152	0.0247	0.0340
	(0.0176)	(0.0244)	(0.0384)	(0.0365)
Inflation rate	0.492***	0.326***	0.212	0.266**
	(0.114)	(0.106)	(0.155)	(0.117)
Real interest rate	0.403***	0.336***	0.253*	0.316***
	(0.0355)	(0.0634)	(0.144)	(0.111)
Social expenditures	-0.489***	0.162	1.479	1.360
	(0.180)	(0.170)	(0.894)	(0.837)
Aged dependency ratio	0.378***	-0.0976	-0.458	-0.386
	(0.121)	(0.166)	(0.376)	(0.325)
Youth dependency ratio	0.0536	-0.200	0.464	0.494**
	(0.131)	(0.124)	(0.278)	(0.186)
Wald test (p-val)	0.000	0.000	0.000	0.000
Hansen test (p-val)	0.570	0.249	–	–
Wooldridge test (p-val)	–	–	0.000	0.000
Arellano-Bond test (p-val)				
1st-order autocorr.	0.044	0.016	–	–
2nd-order autocorr.	0.703	0.585	–	–
3rd-order autocorr.	0.945	0.460	–	–
Observations (No. of countries)	139 (22)	165 (24)	194 (24)	224 (26)

Notes: In column (1) and (2), we instrument the endogenous variables using the first 2 lags in the first-differenced model. We also control for the overall financial development using two different indices: the financial reform index in column (1) and (3), the private credit to GDP ratio in column (2) and (4).

F Characterization of the Equilibrium

In this appendix, we will solve the full model. First, we derive the optimal decision rules for a single household and a single firm, then aggregate their individual choices and characterize the competitive equilibrium by a nonlinear equation system.

F.1 A Single Household's Decision Problem

We now solve the household's problem. Following Wen (2009) by denoting $H_t(a_t, s_t) = (Q_t + D_t) a_t + W_t n_t(a_t, s_t) + s_t$ as the total income of the household in period t, the following proposition shows that the income distribution is degenerate, i.e., $H_t(a_t, s_t)$ does not depend on the idiosyncratic state (a_t, s_t). Moreover, there exists a unique cutoff θ_t^* such that the borrowing constraint (4) binds if and only if $\theta_t \geq \theta_t^*$.

Proposition 4 *There exists a cutoff θ_t^*, such that the optimal consumption follows a trigger strategy:*

$$c_t(a_t, s_t, \theta_t) = \min\left(\frac{\theta_t}{\theta_t^*}, 1\right)[H_t(a_t, s_t) + B_t]. \tag{F.1}$$

In particular, given real wage W_t and real interest rate R_{bt}, individual's total income $H_t(a_t, s_t) \equiv H_t$ and cutoff θ_t^* are jointly determined by

$$\theta_t^* = \beta R_{bt} \frac{H_t + B_t}{W_{t+1}/\psi}, \tag{F.2}$$

$$\psi = \frac{W_t}{H_t + B_t} \Phi(\theta_t^*), \tag{F.3}$$

where $\Phi(\theta_t^*) = \int \max(\theta_t, \theta_t^*) dF(\theta_t)$. Finally, the optimal portfolio choice yields

$$\frac{\theta_t^*}{H_t + B_t} = \beta \frac{R_{bt}}{H_{t+1} + B_{t+1}} \Phi(\theta_{t+1}^*), \tag{F.4}$$

$$R_{bt} = \frac{Q_{t+1} + D_{t+1}}{Q_t}. \tag{F.5}$$

Proof. Prove first that the total wealth $H_t(a_t, s_t) = (Q_t + D_t) a_t + W_t n_t(a_t, s_t) + s_t$ is degenerate. In the second sub-period, the household's consumption, savings, and stock holdings can be written as a function of her wealth $H_t(a_t, s_t)$, liquidity shock θ_t, and the aggregate variables. Let $\lambda_t(a_t, s_t, \theta_t)$ and $\mu_t(a_t, s_t, \theta_t)$ be the Lagrangian multipliers for the budget constraint and borrowing constraint, respectively. The FOC for consumption gives

$$\lambda_t(a_t, s_t, \theta_t) = \frac{\theta_t}{c_t(a_t, s_t, \theta_t)}. \tag{F.6}$$

The optimal bonds holding decision implies

$$\begin{aligned} \frac{\theta_t}{c_t(a_t,s_t,\theta_t)} &= \beta R_{bt} \int \lambda_{t+1}(a_{t+1},s_{t+1},\theta_{t+1})\, dF(\theta_{t+1}) + \mu_t(a_t,s_t,\theta_t) \\ &= \beta R_{bt} \frac{\psi}{W_{t+1}} + \mu_t(a_t,s_t,\theta_t), \end{aligned} \tag{F.7}$$

where the second line has made use of optimal labor supply decision. Define a new variable θ_t^* as

$$\theta_t^*(a_t,s_t) = \beta R_{bt} \frac{\psi}{W_{t+1}} [H_t(a_t,s_t) + B_t]. \tag{F.8}$$

Since $c_t(a_t,s_t,\theta_t) \leq H_t(a_t,s_t) + B_t$, then $\frac{\theta_t}{c_t(a_t,s_t,\theta_t)} \geq \frac{\theta_t^*(a_t,s_t)}{H_t(a_t,s_t)+B_t}$ for $\theta_t \geq \theta_t^*(a_t,s_t)$. By (F.7) and (F.8), we have $\mu_t > 0$. I.e., the borrowing constraint (4) binds, and the household's consumption is thus $H_t(a_t,s_t) + B_t$. If $\mu_t(a_t,s_t,\theta_t) = 0$, then we have

$$\frac{\theta_t}{c_t(a_t,s_t,\theta_t)} = \frac{\theta_t^*(a_t,s_t)}{H_t(a_t,s_t)+B_t}, \tag{F.9}$$

or $c_t(a_t,s_t,\theta_t) = [H_t(a_t,s_t) + B_t] \frac{\theta_t}{\theta_t^*(a_t,s_t)}$. Since $c_t(a_t,s_t,\theta_t) \leq H_t(a_t,s_t) + B_t$, we have $\theta_t \leq \theta_t^*(a_t,s_t)$, which confirms θ_t^* is the cutoff. Finally, using the consumption rule derived above, we can rewrite the optimal labor decision as

$$\frac{W_t}{\psi}\left[\int_{\theta<\theta_t^*(a_t,s_t)} \frac{\theta_t^*(a_t,s_t)}{H_t(a_t,s_t)+B_t} dF(\theta) + \int_{\theta>\theta_t^*(a_t,s_t)} \frac{\theta}{H_t(a_t,s_t)+B_t} dF(\theta)\right] = 1. \tag{F.10}$$

Equations (F.8) and (F.10) jointly determine $\theta_t^*(a_t,s_t)$ and $H_t(a_t,s_t)$. It is evident that $H_t(a_t,s_t)$ and $\theta_t^*(a_t,s_t)$ only depend on aggregate variables in the economy. Thus we let $H_t(a_t,s_t) = H_t$, and $\theta_t^*(a_t,s_t) = \theta_t^*$. Dropping the subscripts from equation (F.8) yields equation (F.2). Writing equation (F.10) more compactly and dropping the subscripts gives equation (F.3). Equations (F.1) to (F.5) are straightforward to obtain. ■

As in Wen (2009), with quasilinear preferences, the household can achieve a target wealth H_t in period t to buffer the idiosyncratic preference shocks by adjusting its labor supply. The quasilinear utility function implies that the marginal disutility of acquiring additional labor income is a constant (equal to $\frac{\psi}{W_t}$) for all households. Since the preference shock is i.i.d., the expected marginal utility from consumption depends only on the total wealth in period t (not on the history of idiosyncratic shocks). The household supplies a level of labor such that the marginal disutility equals the expected gains in the marginal utility, which implies a common target wealth for all households.

In the absence of aggregate uncertainty, the no-arbitrage condition between bonds and equity, (F.5), implies that the stock price Q_t is equal to $(Q_{t+1} + D_{t+1})/R_{bt}$. Comparing with (10), we obtain $R_{bt} =$

$\rho\Lambda_{t+1}/\Lambda_t$, i.e., the risk-free rate is the proper discounting factor for firms. With the decision rules of firms and households in hand, we are now ready to characterize the aggregate equilibrium by a set of nonlinear equations.

F.2 A Single Firm's Decision Problem

We conjecture that the value of a firm with ε_t has the following functional form:

$$J_t(k_t, \varepsilon_t) = v_t(\varepsilon_t)\, k_t, \tag{F.11}$$

where v_t is a to-be-determined variable that depends only on the aggregate states and idiosyncratic investment efficiency ε_t. Define $q_t = \rho\frac{\Lambda_{t+1}}{\Lambda_t}\int v_{t+1}(\varepsilon_{t+1})\, dG(\varepsilon_{t+1})$, which will be proved to be the traditional marginal q. With the conjectured value function, the firm's investment problem becomes

$$v_t(\varepsilon_t)\, k_t = \max_{\{i_t, l_t\}} (R_t k_t - i_t) + q_t\left[(1-\delta)\, k_t + \varepsilon_t i_t\right], \tag{F.12}$$

subject to the liquidity constraint (8) and the collateral constraint

$$l_t \le q_t \xi k_t. \tag{F.13}$$

The following proposition characterizes the individual firm's optimal decisions.

Proposition 5 *There exists a cutoff $\varepsilon_t^* = 1/q_t$, such that the firm's optimal investment decisions follow a trigger strategy:*

$$i_t(k_t, \varepsilon_t) = \begin{cases} (q_t\xi + R_t)\, k_t & \text{if } \varepsilon_t > \varepsilon_t^* \\ 0 & \text{otherwise} \end{cases}. \tag{F.14}$$

In addition, the marginal value of the firm is given by

$$v_t(\varepsilon_t) = R_t + (1-\delta)\, q_t + 1_{\varepsilon_t > \varepsilon_t^*} \times (q_t\xi + R_t)(q_t\varepsilon_t - 1), \tag{F.15}$$

and marginal q, q_t, evolves according to

$$q_t = \rho\frac{\Lambda_{t+1}}{\Lambda_t}\left[R_{t+1} + (1-\delta) q_{t+1} + (q_{t+1}\xi + R_{t+1})\Omega(q_{t+1})\right], \tag{F.16}$$

where $\Omega(q_t) \equiv \int_{\varepsilon_t > \varepsilon_t^*}(q_t\varepsilon_t - 1)\, dG(\varepsilon_t)$ with $\Omega'(q_t) > 0$.

Here, $v_t(\varepsilon_t)$ is the average market value of one unit of capital and q_t is the ex-dividend value of one unit of installed capital, which is the marginal benefit of new investment. Since the cost of investment is 1, the additional gain from investing is positive if $q_t\varepsilon_t - 1 > 0$. In this case, the firm tends to fully employ its borrowing capacity to finance the investment, thus the borrowing constraint binds. The value $v_t(\varepsilon_t)$ consists of three parts as shown on the right hand side of equation (F.15). First, one unit of capital can generate R_t units of operating profit in period t. Second, one unit of capital, after depreciating and paying dividends, can carry $1-\delta$ units of capital to the next period with value $(1-\delta)\,q_t$. Finally, the capital can also be used as collateral. Once the firm decides to invest, with one unit of capital $q_t\xi$ units of loans can be obtained, which increases the expected net benefit from investing by $\Omega\left(q_{t+1}\right)\left(q_{t+1}\xi+R_{t+1}\right)$.

The evolution of q_t, (F.16), comes from the definition of q_t. In fact, after multiplying both sides by K_{t+1}, (F.16) is equivalent to the asset pricing formula (10). To see this, aggregating the investment decision (F.14) over firms gives the total investment

$$I_t = \left[1-G\left(\varepsilon_t^*\right)\right]\left(q_t\xi+R_t\right)K_t. \tag{F.17}$$

Given the definition of aggregate investment, multiplying both sides of (F.16) by K_{t+1} gives

$$q_tK_{t+1} = \rho\frac{\Lambda_{t+1}}{\Lambda_t}\left(R_{t+1}K_{t+1}-I_{t+1}+q_{t+1}K_{t+2}\right). \tag{F.18}$$

Comparing (F.18) with the pricing equation (10) yields $Q_t = q_tK_{t+1}$.

F.3 Aggregation and the Dynamic System

The labor demand of an individual firm, $\tilde{n}_t\left(k_t\right) = \left(\frac{1-\alpha}{W_t}\right)^{\frac{1}{\alpha}}k_t$, implies that the capital-labor ratio is the same across firms, i.e., $\frac{k_t}{\tilde{n}_t(k_t,\varepsilon_t)} = \frac{K_t}{N_t}$. It follows that the aggregate production is given by $Y_t = K_t^{\alpha}N_t^{1-\alpha}$. As a result, the factor prices R_t and W_t are given by $R_t = \alpha\frac{Y_t}{K_t}$ and $W_t = (1-\alpha)\frac{Y_t}{N_t}$, respectively. Aggregating individual consumption function, (F.1), over households yields the aggregate consumption $C_t = \Upsilon\left(\theta_t^*\right)\left(H_t+B_t\right)$, where $\Upsilon\left(\theta_t^*\right) = \int\min(\theta_t/\theta_t^*,1)dF\left(\theta_t\right)$ resembles the propensity to consumption. The $\Upsilon\left(\theta_t^*\right)$ is less than 1 because idiosyncratic preference shocks and borrowing constraints give the households a precautionary motive to accumulate wealth. The equilibrium can now be characterized by the following proposition.

Proposition 6 *The equilibrium path of the model is characterized by the dynamics of eight aggregate variables* $\{C_t, I_t, Y_t, N_t, K_{t+1}, R_{bt}, q_t, \theta_t^*\}$, *which can be solved by a system of eight nonlinear difference* *equations:*

$$\Delta_t = \beta R_{bt}\Delta_{t+1}\Phi(\theta^*_{t+1}), \text{ where } \Delta_t = G(\theta^*_t)\theta^*_t/C_t, \tag{F.19}$$

$$\psi = \Phi(\theta^*_t)\Delta_t(1-\alpha)\frac{Y_t}{N_t}, \tag{F.20}$$

$$Y_t = K_t^\alpha N_t^{1-\alpha}, \tag{F.21}$$

$$Y_t = C_t + I_t, \tag{F.22}$$

$$I_t = [1 - G(1/q_t)](q_t\xi + R_t)K_t, \tag{F.23}$$

$$K_{t+1} = (1-\delta)K_t + \Psi(q_t) I_t, \tag{F.24}$$

$$q_t = \frac{1}{R_{bt}}\left[R_{t+1} + (1-\delta)q_{t+1} + (q_{t+1}\xi + R_{t+1})\Omega(q_{t+1})\right] \tag{F.25}$$

$$C_t = \Upsilon(\theta^*_t)(q_t K_{t+1} + Y_t - I_t + B_t), \tag{F.26}$$

where

$$\Phi(\theta^*_t) = \int \max(\frac{\theta_t}{\theta^*_t}, 1)dF(\theta_t), \ \Upsilon(\theta^*_t) = \int \min(\frac{\theta}{\theta^*_t}, 1)dF(\theta_t),$$

$$\Psi(q_t) = \frac{\int_{\varepsilon > 1/q_t} \varepsilon_t dG(\varepsilon_t)}{1 - G(1/q_t)}, \ \Omega(q_t) = \int_{\varepsilon_t > 1/q_t} (q_t\varepsilon_t - 1)\, dG(\varepsilon_t).$$

The system (F.19) to (F.26) looks quite similar to the standard real business cycle (RBC) model. In particular, equation (F.19) resembles the Euler equation for the bonds decision, in which the term $\Delta_t = \Upsilon(\theta^*_t)\theta^*_t/C_t$ can be treated as marginal utility of consumption. Equation (F.20) describes the labor market equilibrium, in which the extra term $\Phi(\theta^*_t)$ is the wedge introduced by the financial frictions.[22] Equations (F.21) and (F.22) are the same as the production function and resource constraint in the RBC model. Equation (F.24) is the accumulation rule of installed capital. Equation (F.23) is the aggregate investment equation under the collateral constraint. The financial frictions introduce two new variables, marginal q, q_t, and the cutoff of idiosyncratic preference shocks, θ^*_t. These two variables are determined by equations (F.25) and (F.26). Specifically, (F.25) is simply the evolution of q_t, and (F.26) is derived from the aggregate consumption equation.[23]

[22] Recall that $G(\theta^*_t)\theta^*_t/C_t$ resembles marginal utility. Moreover, it is easy to show that $\Phi(\theta^*_t)$ is greater than 1. As the labor decision is made before the realization of θ_t, providing one more unit of labor may increase the total income and thus help to relax the borrowing contraint. Therefore, the wedge $\Phi(\theta^*_t)$ reflects the liquidity premium of labor income.

[23] In particular, according to the definition of $H_t(a_t, s_t)$, the aggregate H_t is given by

$$H_t = (Q_t + D_t)\int (a_t + s_t)\, d\chi(a_t, s_t, \theta_t) + W_t N_t = Q_t + D_t + W_t N_t = q_t K_{t+1} + Y_t - I_t.$$

The second equality comes from the fact that $\int a_t d\chi(a_t, s_t, \theta_t) = 1$ and $\int s_t d\chi(a_t, s_t, \theta_t) = 0$; the third equality comes from the fact that $Q_t = q_t K_{t+1}$, $D_t = R_t K_t - I_t$ and $R_t K_t + W_t N_t = Y_t$. The equation (F.26) is obtained by replacing the H_t by the above equation.

F.4 Stationary Equilibrium

We now derive the stationary equilibrium, in which aggregate variables are time invariant. From (F.23) to (F.24) and jointly with $\varepsilon^* = 1/q$, the marginal q can be solved as a function of R and ξ or $q(R, \xi)$, with $\frac{\partial q}{\partial R} > 0$, $\frac{\partial q}{\partial \xi} < 0$. Notice that $R = \alpha \frac{Y}{K} = \alpha k^{\alpha-1}$, where $k = K/N$, (F.24) gives the capital demand function

$$\alpha k^{\alpha-1} = (R_b - 1) q(R, \xi) + \delta \Psi(q(R, \xi)). \tag{F.27}$$

implying that k is decreasing in R_b and increasing in ξ.

Now we derive the capital supply curve from the household side. (F.19) implicitly defines the cutoff θ^* as an increasing function of R_b. From the aggregate consumption equation (F.26) as well as the definition of $H + B$, we obtain the capital supply curve

$$(1-\alpha) k^{\alpha-1} = \frac{\Upsilon(\theta^*)}{1-\Upsilon(\theta^*)} bq + \left[\frac{1}{1-\Upsilon(\theta^*)} - R_b\right] q. \tag{F.28}$$

It is easy to show that the right hand side of the last equation is decreasing in R_b and ξ and increasing in b. (F.27) and (F.28) determine the equilibrium k and R_b. It is easy to see that a higher ξ shifts the (F.27) curve rightward, thus raises equilibrium k, while a higher b shifts the (F.28) curve leftward, thus reducing equilibrium k. Notice that under the condition $\frac{1-G(\varepsilon^*)}{g(\varepsilon^*)\varepsilon^*} > 1$, it can be shown that the savings rate s $(= I/Y)$ is strictly increasing in k.[24] Therefore, (F.27) and (F.28) can be transformed to

$$\mathbf{KD}(s, R_b; \xi) = 0, \tag{F.29}$$

$$\mathbf{KS}(s, R_b; b, \xi) = 0. \tag{F.30}$$

And the equilibrium saving rate s is decreasing in b and increasing in ξ. It is worth noting that the above analysis is still valid if only one type of financial friction presents. In particular, if only households are financially constrained, the $\mathbf{KD}(s, R_b; \xi)$ will degenerate to $s - \frac{\alpha\delta}{R_b - 1 + \delta}$; if only firms are financially constrained, the $\mathbf{KS}(s, R_b; b, \xi)$ will degenerate to $R_b - 1/\beta$.

[24] The condition $\frac{1-G(\varepsilon^*)}{g(\varepsilon^*)\varepsilon^*} > 1$ guarantees $\frac{\int_{\varepsilon \geq \varepsilon^*} \varepsilon dG(\varepsilon)}{\varepsilon^*[1-G(\varepsilon^*)]}$ is decreasing in ε^* or increasing in q. Equations (F.23) and (F.24) imply that the savings rate s satisfies $s = \alpha\delta \left[\frac{\delta}{1-G(\varepsilon^*)} - \frac{\Gamma(\varepsilon^*)}{[1-G(\varepsilon^*)]\varepsilon^*}\xi\right]^{-1}$. Therefore, s is increasing (decreasing) in ε^* (q) and increasing in ξ. Furthermore, recall that we have $\partial q/\partial R > 0$ or equivalently $\partial q/\partial k < 0$, consequently s is increasing in k.