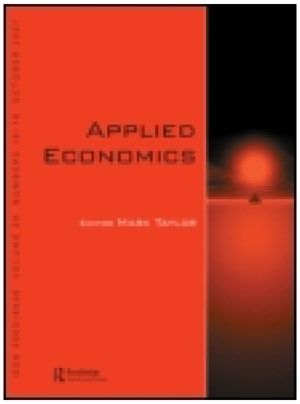

Applied Economics

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/raec20

Univariate unobserved-component model with a nonrandom-walk permanent component

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To cite this article: Zhiwei Xu (2013) Univariate unobserved-component model with a nonrandom-walk permanent component, Applied Economics, 45:33, 4733-4737, DOI: 10.1080/00036846.2013.799756

To link to this article: http://dx.doi.org/10.1080/00036846.2013.799756

Applied Economics, 2013
Vol. 45, No. 33, 4733–4737, http://dx.doi.org/10.1080/00036846.2013.799756

Univariate unobserved-component model with a nonrandom-walk permanent component

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In this article, we revisit the univariate unobserved-component (UC) model of the US GDP by relaxing the traditional random-walk assumption of the permanent component. Since our general UC model is unidentified, we investigate the upper bound of the contribution of the transitory component, and find the GDP fluctuation is dominated by the permanent component.

Keywords: unobserved-component model; random-walk assumption; permanent and transitory shocks

JEL Classification: C22; C49; E32

I. Introduction

Morley *et al*. (2003) study the equivalence of univariate unobserved-component (UC) model and the Beveridge–Nelson (BN) (1981) decomposition. They conclude that the permanent component of the US GDP extracted by the UC model is exactly the same as the BN trend. The innovations of the two (permanent and transitory) components are highly negatively correlated (further discussions about this point can be found in a recent paper by Oh *et al*. (2008)). The nonorthogonality of the two innovations is mainly caused by the random-walk assumption imposed on the permanent component, see Nagakura (2008) for the formal discussion. In this article, we relax the random-walk assumption by allowing the permanent component to follow a general unit root process. Under our assumption, the real GDP can be decomposed into two orthogonal parts so that the impulse responses to permanent and transitory shocks can be implemented. Since our generalization of the random-walk assumption increases the parameter set of the UC model, the model becomes unidentified. However, we can investigate the upper bound of the contribution of the transitory component to GDP and study the dynamics of this extreme case by implementing impulse response and variance decomposition. We find that the transitory component explains less than 35% of output volatility; therefore, the permanent component is the main source of the GDP fluctuation.

II. The UC Model

Our modified UC representation takes the form

$$
\begin{aligned}
y_t &= g_t + c_t \\
g_t &= \mu + g_{t-1} + \frac{\Theta_{q_1}(L)}{\Phi_{p_1}(L)}\eta_t, \eta \sim i.i.d\ N(0, \sigma_\eta^2) \qquad (1) \\
c_t &= \frac{\Theta_{q_2}(L)}{\Phi_{p_2}(L)}\varepsilon_t,\ \varepsilon \sim i.i.d\ N(0, \sigma_\varepsilon^2)
\end{aligned}
$$

where $\{y_t\}$ is the log real GDP, and $\{g_t\}$ is an unobserved permanent component with a unit root (i.e. its first difference is an autoregressive-moving-average (ARMA) (p_1, q_1) process with drift μ). The unobserved transitory component $\{c_t\}$ is a stationary ARMA (p_2, q_2) process. Moreover, we assume the two innovations satisfy

$$cov(\eta_t, \varepsilon_{t\pm k}) = \begin{cases} \sigma_{\eta\varepsilon} & \text{for } k = 0 \\ 0 & \text{otherwise} \end{cases}$$

The parameters under interest include the mean growth rate μ, and the coefficients of the two ARMA process, $\{\Phi_{p_1}(L), \Theta_{q_1}(L), \Phi_{p_2}(L), \Theta_{q_2}(L), \sigma_\eta, \sigma_\varepsilon, \sigma_{\eta\varepsilon}\}$.

Writing the model (1) more compactly gives the ARIMA representation of y_t,

$$\begin{aligned}\Phi_{p_1}(L)\Phi_{p_2}(L)\Delta y_t = {} & \Phi_{p_1}(1)\Phi_{p_2}(1)\mu \\ & + \Phi_{p_2}(L)\Theta_{q_1}(L)\eta_t \\ & + (1-L)\Phi_{p_1}(L)\Theta_{q_2}(L)\varepsilon_t\end{aligned} \tag{2}$$

This expression implies we can recover the parameters of the UC model by estimating the growth rate of GDP as a ARIMA process. Here, we follow the strategy of Morley *et al*. (2003) to estimate the GDP as an ARIMA (2,1,2) process:[1]

$$\begin{aligned}(1 - \phi_1 L - \phi_2 L^2)\Delta y_t = {} & (1 - \phi_1 - \phi_2)\mu^* \\ & + (1 + \theta_1 L + \theta_2 L^2)u_t\end{aligned} \tag{3}$$

Table 1 reports the estimated results. Note that γ_j are the j-th order auto-covariance of MA part of the ARIMA process, and μ^*, σ_u and γ_j are in percentages. The data used are US quarterly real GDP from 1948:Q1 to 2008:Q1.

The absence of real roots in AR part indicates that the polynomial $(1 - \phi_1 L - \phi_2 L^2)$ cannot be factored further. This fact induces us to determine the form of $\Phi_{p_1}(L)$ and $\Phi_{p_2}(L)$ only in two alternative ways: $\Phi_{p_1}(L) = 1$, $\Phi_{p_2}(L) = (1 - \phi_1 L - \phi_2 L^2)$ or $\Phi_{p_1}(L) = \Phi_{p_2}(L) = (1 - \phi_1 L - \phi_2 L^2)$.[2] Obviously, the first case is just the specification in Morley *et al*. (2003) in which permanent component g_t is a random walk. And the second case is the one we want to discuss in which g_t is a general ARIMA (2,1,2) process.

Table 1. Maximum likelihood estimates for ARIMA (2,1,2)

	Estimate	SE
Drift μ*	0.8264	(0.0765)
ϕ_1	1.3638	(0.1227)
ϕ_2	−0.7616	(0.0843)
θ_1	−1.1039	(0.1319)
θ_2	0.5976	(0.1004)
σ_u	0.9068	(0.0311)
AR roots (inverted)	$0.8954 \pm 0.7151i$	
γ_0	2.1184	
γ_1	−1.4505	
γ_2	−0.4915	
Log likelihood	−317.2356	
Long-run effect of u_t	1.2411	

Once $\Phi_{p_1}(L)$ and $\Phi_{p_2}(L)$ are determined, we can find the form of MA polynomials $\Theta_{q_1}(L)$ and $\Theta_{q_2}(L)$. In particular, to ensure the right-hsnd side RHS of Equation 2 be a MA(2) process, $\Theta_{q_1}(L)$ and $\Theta_{q_2}(L)$ can at most take the form of $(1 + \psi_1 L + \psi_2 L^2)$ and $(1 + \theta L)$, respectively. Now the parameters of interest are $\{\psi_1, \psi_2, \theta, \sigma_\eta, \sigma_\varepsilon, \sigma_{\eta\varepsilon}\}$,[3] and the representation (2) is reduced to

$$\begin{aligned}(1 - \phi_1 L - \phi_2 L^2)\Delta y_t = {} & (1 - \phi_1 - \phi_2)\mu \\ & + \left(1 + \psi_1 L + \psi_2 L^2\right)\eta_t \\ & + (1 - L)(1 + \theta L)\varepsilon_t\end{aligned} \tag{4}$$

Remember that we have estimated the auto-covariances of the RHS of the last equation from the data, see $\{\gamma_0, \gamma_1, \gamma_2\}$ in Table 1. Equate these moments to their counterparts in Equation 4 and after some algebra, we get three equations for six parameters $\{\psi_1, \psi_2, \theta, \sigma_\eta, \sigma_\varepsilon, \sigma_{\eta\varepsilon}\}$:

$$\begin{aligned}\sigma_\eta^2 &= \frac{\gamma_0 + 2\gamma_1 + 2\gamma_2}{(1 + \psi_1 + \psi_2)^2} \\ \sigma_\varepsilon^2 &= \frac{-2(1 + \psi_1\theta - \psi_1 - \psi_2\theta)(\gamma_2 - \psi_2\sigma_\eta^2) - (\theta - \psi_2)[\gamma_0 - (1 + \psi_1^2 + \psi_2^2)\sigma_\eta^2]}{2\theta(1 + \psi_1\theta - \psi_1 - \psi_2\theta) - 2(\theta - \psi_2)(1 - \theta + \theta^2)} \\ \sigma_{\eta\varepsilon} &= \frac{\theta[\gamma_0 - (1 + \psi_1^2 + \psi_2^2)\sigma_\eta^2] + 2(1 - \theta + \theta^2)(\gamma_2 - \psi_2\sigma_\eta^2)}{2\theta(1 + \psi_1\theta - \psi_1 - \psi_2\theta) - 2(\theta - \psi_2)(1 - \theta + \theta^2)}\end{aligned} \tag{5}$$

[1] Oh *et al*. (2008) also recommend this specification. They find that ARIMA (2,1,2) is preferred by the Akaike information criterion (AIC) and ARIMA (1,1,0) is preferred by the Bayesian information criterion (BIC). However, the latter specification is not able to capture the periodical behaviour of output due to its oversimplified structure.

[2] The setting $\Phi_{p_1}(L) = (1 - \phi_1 L - \phi_2 L^2)$, $\Phi_{p_2}(L) = 1$ is infeasible, since this will make the order of MA part of Δy_t (the RHS of Equation 2) exceed 2.

[3] The mean growth rate μ is just the same as that in ARIMA representation.

The MA(2) process has only three auto-variances, but we have six unknown parameters. This implies our UC model is unidentified.

In order to obtain two structural (or orthogonal) shocks, we need to set $\sigma_{\eta\varepsilon}$ to be zero. The reader may ask whether this restriction is feasible,[4] since in Morley *et al*. (2003), when permanent component is a random walk, two innovations are always highly negatively correlated. In fact, as long as the long-run effect (see the last row in Table 1) in the ARIMA representation of GDP is larger than 1, the orthogonality restriction in our modified UC model is always feasible. A formal mathematical proof can be found in Corollary 1 of Nagukara (2008).

To learn the relationships of the unknown parameters, one method is to solve three of them as functions of the other two. Unfortunately, equation system 5 is nonlinear and fairly complicated, we cannot solve it in a closed form. So we resort to numerical method. Figure 1 plots $\{\psi_1, \sigma_\eta, \sigma_\varepsilon\}$ as functions of ψ_2 when $\theta = 0$. For other values of θ in (−1,1), the pattern changes little. In addition, to ensure Δg_t be invertible and σ_ε^2 be always positive, ψ_2 must be in the range around 0.6–1.

One thing worth noting in Fig. 1 is that $\{\psi_1, \sigma_\eta, \sigma_\varepsilon\}$ are monotonic functions of ψ_2, and the monotonicity does not change for different θ. Furthermore, the SD of transitory shock ε_t reaches its maximum when ψ_2 approaches to 1. Since σ_ε is a continuous function of ψ_2 and θ, without loss of generality, we fix $\psi_2 = 1$ for different θ to find the largest transitory component (in terms of variance) in our modified UC model. Figure 2 plots σ_ε against θ, when $\psi_2 = 1$. From the figure, we can see that σ_ε reaches its unique maximum of 0.4442 at $\theta = -0.63$.

The above analysis implies that our UC model can be just identified, if the transitory and permanent components are forced to be orthogonal and the volatility of transitory component reaches its upper bound. In Section III, we will study the dynamic features of the two components under the above identification method and compare the results with those obtained by using the Blanchard–Quah (BQ) (1989) decomposition.

III. Dynamics

The largest possible variance of the transitory component $\{c_t\}$ has SD of 0.4442 when setting $\theta = -0.63$ and $\psi_2 = 1$. The remaining parameters ψ_1 and σ_η can be solved directly from equation system 5. In particular, we have $\psi_1 = -1.2612$ and $\sigma_\eta = 0.6059$.[5] Since both BQ (1989)[6] and our UC model implement orthogonal decomposition with a general unit root permanent component, we can use impulse responses and variance decomposition to compare our results with theirs. To ensure consistency (i.e. GDP in the bivariate BQ decomposition must also follow an ARIMA (2,1,2) process), we estimate a two-variable vector autoregression (VAR) system with GDP growth and unemployment rate as a vector ARMA (1,1) process. We use RATS 7.0 (Estima, Evanston, IL, USA) to conduct the estimation.

Figure 3 plots the impulse responses of GDP to a one SD permanent and transitory shock, respectively.[7] In particular, under the permanent shock η_t (the left graph), output in our UC model has a larger and periodic response

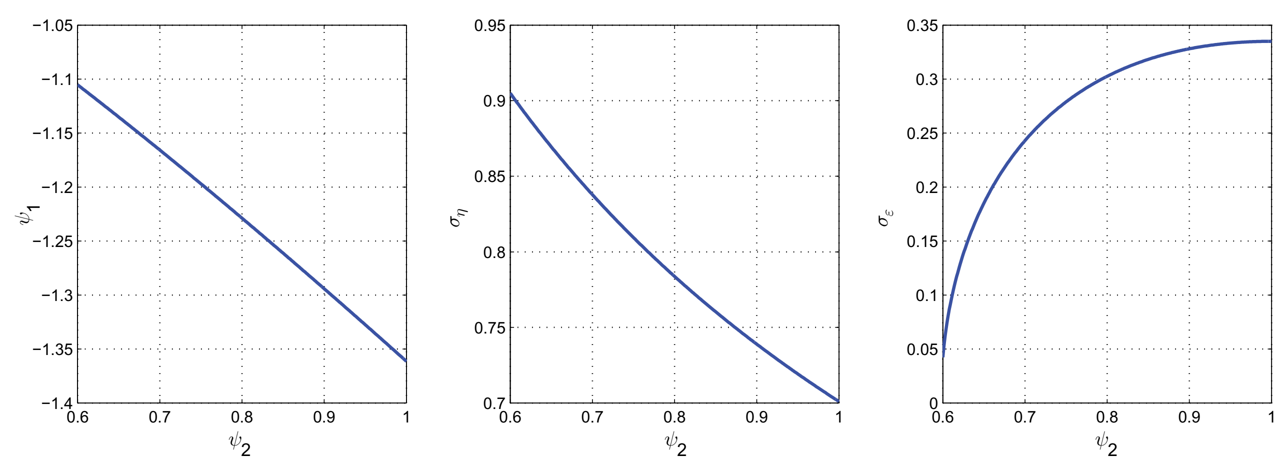

Fig. 1. The relationship between $\{\psi_1, \sigma_\eta, \sigma_\varepsilon\}$ and ψ_2 when $\theta = 0$

[4] Here, 'feasible' means that equation system 5 always has solution when $\sigma_{\eta\varepsilon} = 0$.
[5] The parameters $\{\sigma_\varepsilon, \sigma_\eta, \psi_1\}$ are statistically significant, we calculate their *t*-statistics by bootstrapping method, but not reported here.
[6] In their paper, Blanchard–Quah (BQ) decompose GDP based on a structural bivariate VAR system of (ΔGDP, unemployment rate). They just identify the model by imposing a long-run restriction on the transitory component.
[7] The dashed lines are 95% bootstrapped confidence interval computed (200 replications) by Hall's percentile interval.

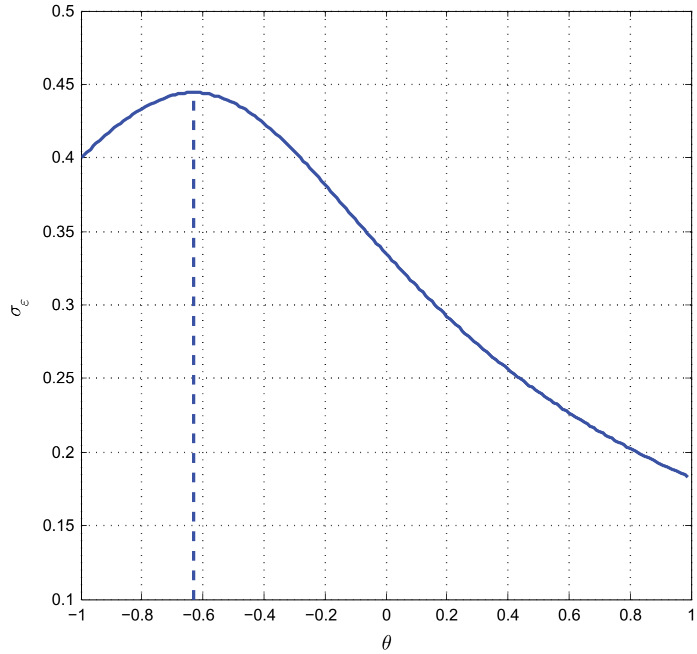

Fig. 2. The maximum of σ_ε for different θ in (–1,1)

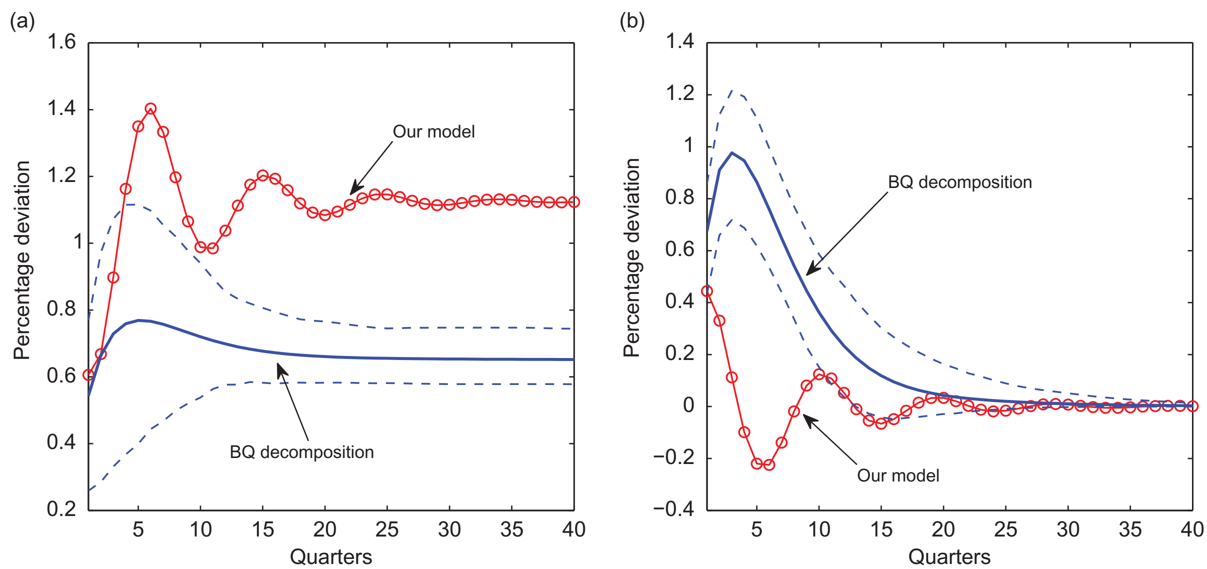

Fig. 3. Impulse responses of GDP to different shocks. (a) Response to permanent shock, (b) Response to transitory shock

compared with that obtained by the BQ method. The maximum response climbs to the peak after six quarters. The long-run effect of the permanent shock is also significantly larger (about 1.1), while under the BQ decomposition this value is only around 0.6. Under the transitory shock ε_t (the right graph), output movement in our model dies out quickly, while under the BQ decomposition the response is much larger and more persistent.

To see the relative importance of two shocks to the GDP volatility, Table 2 reports the variance decomposition, i.e. the proportion of fluctuations due to transitory shock ε_t in different forecasting horizons.

The numbers in parentheses are 95% confidence intervals. Even though these error bands of the BQ decomposition are large, contribution of transitory shocks to GDP are significantly lower in our model even compared with the

Table 2. Variance decompositions in different models

Percentage of forecasting error due to transitory shock		
Horizon (quarters)	Our model ($\psi_2 = -1$, $\theta = -0.63$)	BQ decomposition
1	34.96	61.06 (25.92, 91.38)
2	27.17	64.11 (29.16, 93.24)
3	16.29	62.04 (27.50, 93.23)
4	9.88	59.96 (26.40, 92.21)
8	4.26	54.63 (25.94, 81.71)
12	3.28	50.56 (26.08, 70.12)
40	0.98	27.00 (13.85, 38.95)

Table 3. ARIMA (2,1,2) implied by VARMA (1,1)

AR part		MA part				
ϕ_1	ϕ_1	θ_1	θ_2	σ	Long-run effect of innovation	Log likelihood
1.4863	–0.5564*	–1.1969	0.2461*	0.9149	0.7193	–317.8866

Note: Asterisks indicate the value is significantly different from the univariate ARMA (2,2) used in the UC model.

lower bound of the BQ decomposition (except for the impact period). That is, our model attributes most fluctuations of output to permanent shock; the transitory component is less important.

To see what may have caused these discrepancies in the two different approaches, we compare the data-generating processes of output implied by these two estimations. Since we estimate the bivariate system of BQ decomposition as a vector autoregressive-moving-average (VARMA) (1,1) process, the growth rate of GDP can be recovered as an ARMA (2,2) process. Table 3 (in comparison with Table 1) lists the implied parameters under the VARMA. Clearly, these different values implied by the VARMA (1,1) and the univariate ARMA (2,2) induce a much smaller long-run effect. This explains why the permanent shock in the BQ decomposition has smaller long-run effect than what we obtain in the UC model.[8]

IV. Conclusions

This article re-examines the UC method of decomposition of GDP by relaxing the random-walk assumption made in the existing UC literature. Based on this generalization, we are able to decompose GDP into two orthogonal components: permanent and transitory. The orthogonality allows us to conduct impulse response analysis and variance decompositions. We find that the permanent component explains the bulk of GDP fluctuations, in sharp contrast to the conclusion reached by Blanchard and Quah (1989).

Acknowledgements

I am grateful to Yi Wen for his comments and continuous encouragement. I also thank Daqing Luo, Pengfei Wang and seminar participants at Shanghai University of Finance and Economics, Tsinghua University. Of course, all errors are mine.

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[8] In fact, this point can be easily seen from a spectrum perspective: the spectrum of growth rate of GDP shares the same value with growth rate of permanent component at zero frequency, and this value is just the squared long-run effect multiplying the variance of innovation in ARIMA process.